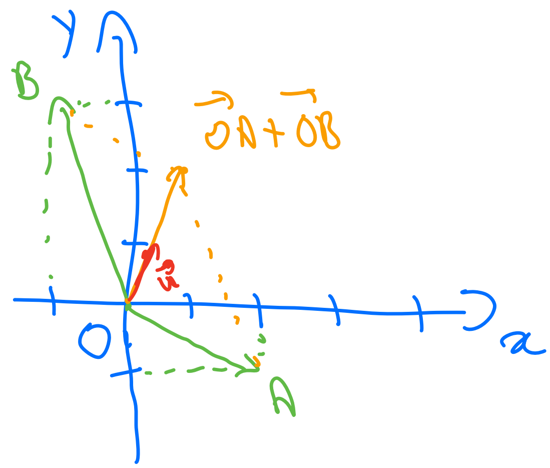
Exo 1

$\vec{OA}\begin{vmatrix}2\\-1\\0\end{vmatrix}$ $\quad \vec{OB}\begin{vmatrix}-1\\3\\0\end{vmatrix}$

1)



$\|\vec{OA}\| = \sqrt{2^2+(-1)^2} = \sqrt{5}$

$\|\vec{OB}\| = \sqrt{(-1)^2+3^2} = \sqrt{10}$

$\vec{OA}+\vec{OB}\begin{vmatrix}1\\2\\0\end{vmatrix}$

$\|\vec{OA}+\vec{OB}\| = \sqrt{5}$

2) $\vec{u} = \dfrac{\vec{OA}+\vec{OB}}{\|\vec{OA}+\vec{OB}\|} = \begin{vmatrix}1/\sqrt{5}\\2/\sqrt{5}\\0\end{vmatrix}$

3) $\vec{OA}\cdot\vec{OB} = \begin{vmatrix}2\\-1\\0\end{vmatrix}\cdot\begin{vmatrix}-1\\3\\0\end{vmatrix} = -5$

$\vec{OA}\cdot\vec{OB} = \|\vec{OA}\|\|\vec{OB}\|\cos(\theta) \Rightarrow \cos(\theta) = \dfrac{-5}{\sqrt{5}\times\sqrt{10}} = -\dfrac{1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$

$\Rightarrow \theta = \dfrac{3\pi}{4}$ $\quad (\theta = 135°)$

4) $\vec{u}$ dans le plan $(xOy) \Leftrightarrow \vec{u} \perp \vec{e_3}$

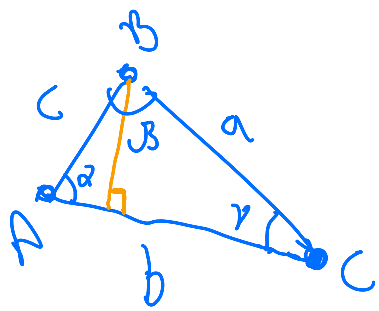
$\cos(\vec{u}, \vec{e_1}) = \dfrac{\vec{u}\cdot\vec{e_1}}{\|\vec{u}\|\|\vec{e_1}\|} = \dfrac{1}{\sqrt{5}} \Rightarrow \text{Arccos}\left(\dfrac{1}{\sqrt{5}}\right) = 1{,}107 \text{ rad}$

$\Rightarrow 63{,}4°$

$$\cos(\vec{u}, \vec{e_2}) = \frac{\vec{u}.\vec{e_2}}{\|\vec{u}\|\|\vec{e_2}\|} = \frac{2}{\sqrt{5}} \Rightarrow 26,6°$$

5) $\vec{OA} \wedge \vec{OB} = \begin{vmatrix} 2 \\ -1 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} -1 \\ 3 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 5 \end{vmatrix}$ $\quad \|\vec{OA} \wedge \vec{OB}\| = 5$

6) Aire d'un triangle quelconque.



$$\mathcal{A} = \frac{b \times h}{2}$$

Loi des sinus : $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

$$\Rightarrow h = a\sin\gamma = a\sin\beta \times \frac{c}{b}$$

$$\Rightarrow \mathcal{A} = \frac{ac\sin\beta}{2} = \frac{\|\vec{OA}\|\|\vec{OB}\|\sin\theta}{2} = \frac{1}{2}\|\vec{OA} \wedge \vec{OB}\|$$

Exo 2

$\vec{U} \begin{vmatrix} 0 \\ 3 \\ 1 \end{vmatrix}$ $\quad \vec{V}$

1) $\vec{U}.\vec{V} = 1 = \|\vec{U}\|\|\vec{V}\|\cos(\phi)$

$\|\vec{U}\| = \sqrt{3^2+1^2} = \sqrt{10}$

$\|\vec{V}\| = \sqrt{1^2+(-2)^2} = \sqrt{5}$

$\cos(\phi) = \frac{1}{5\sqrt{2}}$ $\quad$ $\mathrm{Arccos}\left(\frac{1}{5\sqrt{2}}\right) = 1{,}429$ rad

$\phi = 81{,}9°$

2) $* \vec{u}$ $\quad \alpha_{\vec{u}} = 0$ $\qquad * \vec{v}$ $\quad \alpha_{\vec{v}} = 0$

$\beta_{\vec{u}} = \frac{3}{\sqrt{10}}$ $\qquad \beta_{\vec{v}} = \frac{1}{\sqrt{5}}$

$\gamma_{\vec{u}} = \frac{1}{\sqrt{10}}$ $\qquad \gamma_{\vec{v}} = \frac{-2}{\sqrt{5}}$

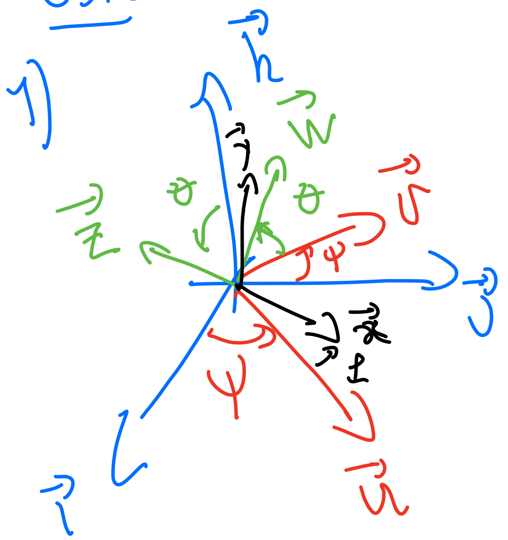
3) $\vec{w} = \begin{vmatrix} 0 \\ 3 \\ 1 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ 1 \\ -2 \end{vmatrix} = \begin{vmatrix} -7 \\ 0 \\ 0 \end{vmatrix}$ $\qquad \|\vec{w}\| = 7$

calcul direct

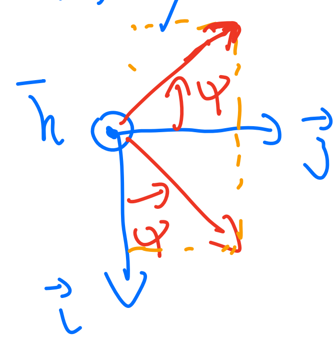
$\|\vec{w}\| = \|\vec{u}\| \, \|\vec{v}\| \sin(\phi) = 7$ avec application numérique

Exo 3

1)



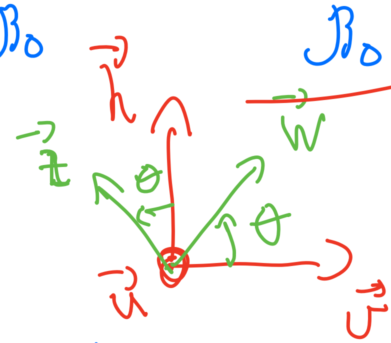
2) $* \mathcal{B}_1 (\vec{u}, \vec{v}, \vec{h})$?



$$\vec{u}\begin{vmatrix} \cos\psi \\ \sin\psi \\ 0 \end{vmatrix}_{\mathcal{B}_0} \quad \vec{v}\begin{vmatrix} -\sin\psi \\ \cos\psi \\ 0 \end{vmatrix}_{\mathcal{B}_0} \quad \vec{h}\begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix}_{\mathcal{B}_0}$$

$* \mathcal{B}_1$

$$\vec{u}\begin{vmatrix} \cos\psi \\ \sin\psi \\ 0 \end{vmatrix}_{\mathcal{B}_0}$$



$$\vec{w} = \cos\theta\,\vec{v} + \sin\theta\,\vec{h}$$

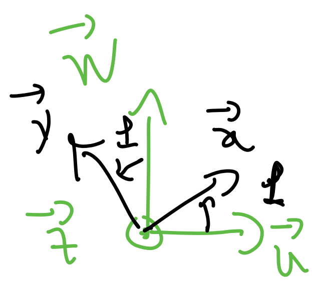
$$\vec{z} = -\sin\theta\,\vec{v} + \cos\theta\,\vec{h}$$

$$\vec{w}\begin{vmatrix} \cos\theta \times -\sin\psi \\ \cos\theta \times \cos\psi \\ \sin\theta \end{vmatrix}_{\mathcal{B}_0}$$

$$\vec{z}\ \left|\begin{array}{l} -\sin\theta \times -\sin\psi \\ -\sin\theta \times \cos\psi \\ \cos\theta \end{array}\right.$$
$\mathcal{B}_0$

$$\vec{u}\ \left|\begin{array}{l} \cos\psi \\ \sin\psi \\ 0 \end{array}\right.\qquad \vec{w}\ \left|\begin{array}{l} -\cos\theta\sin\psi \\ \cos\theta\cos\psi \\ \sin\theta \end{array}\right.\qquad \vec{z}\ \left|\begin{array}{l} \sin\theta\sin\psi \\ -\sin\theta\cos\psi \\ \cos\theta \end{array}\right.$$
$\mathcal{B}_0$ $\qquad\mathcal{B}_0$ $\qquad\mathcal{B}_0$

$*\ \mathcal{B}_3$



$$\vec{x} = \cos\phi\,\vec{u} + \sin\phi\,\vec{w}$$
$$\vec{y} = -\sin\phi\,\vec{u} + \cos\phi\,\vec{w}$$

$$\vec{x}\ \left|\begin{array}{l} \cos\phi\cos\psi - \sin\phi\cos\theta\sin\psi \\ \cos\phi\sin\psi + \sin\phi\cos\theta\cos\psi \\ \sin\phi\sin\theta \end{array}\right.$$
$\mathcal{B}_0$

$$\vec{y} \begin{vmatrix} -\sin\varphi\cos\psi - \cos\varphi\cos\theta\sin\psi \\ -\sin\varphi\sin\psi + \cos\varphi\cos\theta\cos\psi \\ \cos\varphi\sin\theta \end{vmatrix}_{B_0}$$

$$\vec{z} \begin{vmatrix} \sin\theta\sin\psi \\ -\sin\theta\cos\psi \\ \cos\theta \end{vmatrix}$$

3) $$\vec{i} \wedge \vec{u} = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} \cos\psi \\ \sin\psi \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ \sin\psi \end{vmatrix}$$

$$\vec{i} \wedge \vec{v} = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} -\sin\psi \\ \cos\psi \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ \cos\psi \end{vmatrix}$$

$$\vec{j} \wedge \vec{z} = \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} \sin\theta \sin\psi \\ -\sin\theta \cos\psi \\ \cos\theta \end{vmatrix} = \begin{vmatrix} \cos\theta \\ 0 \\ -\sin\theta \sin\psi \end{vmatrix}$$

$$\vec{h} \wedge \vec{x} = \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} \wedge \begin{vmatrix} \cos\phi \cos\psi - \sin\phi \cos\theta \sin\psi \\ \cos\phi \sin\psi + \sin\phi \cos\theta \cos\psi \\ \sin\phi \sin\theta \end{vmatrix}$$

$$\vec{h} \wedge \vec{x} \begin{vmatrix} -\cos\phi \sin\psi - \sin\phi \cos\theta \cos\psi \\ \cos\phi \cos\psi - \sin\phi \cos\theta \sin\psi \\ 0 \end{vmatrix}$$

$(\vec{z} \wedge \vec{y}) \wedge \vec{i}$?

$$\begin{vmatrix} \sin\theta \sin\psi \\ -\sin\theta \cos\psi \\ \cos\theta \end{vmatrix} \wedge \begin{vmatrix} -\sin\phi \cos\psi - \cos\phi \cos\theta \sin\psi \\ -\sin\phi \sin\psi + \cos\phi \cos\theta \cos\psi \\ \cos\phi \sin\theta \end{vmatrix}$$

$$
= \begin{pmatrix}
-\cos\varphi\,\underline{\sin^2\theta}\cos\psi + \sin\varphi\cos\theta\sin\psi - \cos\varphi\,\underline{\cos^2\theta}\cos\psi \\
-\sin\varphi\cos\theta\cos\psi - \cos\varphi\,\underline{\cos^2\theta}\sin\psi - \cos\varphi\,\underline{\sin^2\theta}\sin\psi \\
-\sin\varphi\sin\theta\sin^2\psi + \cos\varphi\,\cancel{\cos\theta\sin\theta}\cos\psi\sin\psi \\
-\sin\varphi\sin\theta\cos^2\psi - \cos\varphi\,\cancel{\cos\theta\sin\theta}\cos\psi\sin\psi
\end{pmatrix}
$$

$$
\vec{z} \wedge \vec{y} \begin{pmatrix}
-\cos\varphi\cos\psi + \sin\varphi\cos\theta\sin\psi \\
\sin\varphi\cos\theta\cos\psi - \cos\varphi\sin\psi \\
-\sin\varphi\sin\theta
\end{pmatrix}
$$

$$
(\vec{z} \wedge \vec{y}) \wedge \vec{i} = \begin{pmatrix}
-\cos\varphi\cos\psi + \sin\varphi\cos\varphi\sin\psi \\
\sin\varphi\cos\theta\cos\psi - \cos\varphi\sin\psi \\
-\sin\varphi\sin\theta
\end{pmatrix} \wedge \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}
$$

$$
\boxed{(\vec{z} \wedge \vec{y}) \wedge \vec{i} = \begin{pmatrix}
0 \\
-\sin\varphi\sin\theta \\
-\sin\varphi\cos\theta\cos\psi + \cos\varphi\sin\psi
\end{pmatrix}}
$$

4) $\vec{w} = \alpha \vec{k} + \beta \vec{u} + \delta \vec{z}$

Dans B_0 $(\vec{i}, \vec{j}, \vec{k})$

$$\vec{w} \left| \begin{array}{l} \beta \cos\psi + \delta \sin\theta \sin\psi \\ \beta \sin\psi - \delta \sin\theta \cos\psi \\ \alpha + \delta \cos\theta \end{array} \right._{B_0}$$

Dans B_1 $(\vec{u}, \vec{v}, \vec{k})$ $\quad \vec{z} = -\sin\theta \, \vec{v} + \cos\theta \, \vec{k}$

$$\vec{w} \left| \begin{array}{l} \beta \\ -\delta \sin\theta \\ \alpha + \delta \cos\theta \end{array} \right._{B_1}$$

Dans $B_2\,(\vec{u}, \vec{w}, \vec{z})$

$\vec{W} = \alpha \vec{h} + \beta \vec{u} + \delta \vec{z}$

$\vec{h} \cdot \vec{u} = 0$

$\vec{h} \cdot \vec{w} = \sin\theta$

$\vec{h} \cdot \vec{z} = \cos\theta$

$\vec{h} = \sin\theta\, \vec{w} + \cos\theta\, \vec{z}$

$$\Rightarrow \boxed{\vec{W} \left| \begin{array}{l} \beta \\ \alpha \sin\theta \\ \delta + \alpha\cos\theta \end{array} \right.}_{B_2}$$

Dans $B_3\,(\vec{x}, \vec{y}, \vec{z})$

$\vec{W} = \alpha \vec{h} + \beta \vec{u} + \delta \vec{z}$

$\vec{h} \cdot \vec{x} = \sin\varphi \sin\theta$

$\vec{h} \cdot \vec{y} = \cos\varphi \sin\theta$

$\vec{h} \cdot \vec{z} = \cos\theta$

$$\vec{u}\cdot\vec{x} = \cos\varphi\cos^2\psi - \sin\varphi\,\cancel{\cos\theta}\cos\psi\sin\psi$$
$$+\cos\varphi\sin^2\psi + \sin\varphi\,\cancel{\cos\theta}\cos\psi\sin\psi$$

$$\vec{u}\cdot\vec{x} = \cos\varphi$$

$$\vec{u}\cdot\vec{y} = -\sin\varphi\cos^2\psi - \sin\cancel{\varphi}\cos\theta\cos\psi\sin\psi$$
$$-\sin\varphi\sin^2\psi + \sin\cancel{\varphi}\cos\theta\cos\psi\sin\psi$$

$$\vec{u}\cdot\vec{y} = -\sin\varphi$$

$$\boxed{\begin{array}{c|l} \vec{W} & \alpha\sin\varphi\sin\theta + \beta\cos\varphi \\ & \alpha\cos\varphi\sin\theta - \beta\sin\varphi \\ & \alpha\cos\theta + \delta \\ \beta_3 & \end{array}}$$