#### **ORIGINALARBEITEN/ORIGINALS**



# Reducing mass while improving the operational behavior: form optimization of planetary gearbox housings

Julian Theling<sup>1,2</sup> · Jens Brimmers<sup>1</sup> · Christian Brecher<sup>1</sup>

Received: 1 December 2020 / Accepted: 12 February 2021 © The Author(s) 2021

#### Abstract

The optimization of load sharing between planets is one of the most important goals in planetary gearbox design. Unevenly distributed load will cause locally higher flank pressures and therefore, less durability of gears and bearings. Furthermore, unevenly distributed or fluctuating loads can cause excitations in the gear mesh and structural vibrations. The load sharing in planetary gear stages depends on the individual stiffness conditions in each mesh position. The stiffness is not only influenced by the gear geometry but also by the surrounding structural elements like shafts, housings and torque arms. In wind industry these components are often designed very stiff in order to reduce their effect on the operational behavior. Within this paper, a method is presented, which allows combining the structural optimization process with a tooth contact analysis for planetary gearboxes. By means of this combined approach, it is possible to optimize the housing structure of the ring gear in terms of mass reduction while keeping the operational behavior in focus. With a weighted design objective function, it is possible to decide whether the main objective should be load distribution, excitation behavior, low mass or a balanced design.

# Reduktion der Masse bei Verbesserung des Einsatzverhaltens: Formoptimierung von Planetengetriebegehäusen

#### Zusammenfassung

Die Optimierung der Lastaufteilung zwischen den einzelnen Planeten ist eines der Hauptziele in der Auslegung von Planetengetrieben. Ungleichmäßig aufgeteilte Lasten bedingen unter anderem höhere Zahnflankenpressungen und damit eine geringere Lebensdauer der Verzahnungen und der Lager. Zusätzlich können ungleichmäßig verteilte oder zeitlich variierende Lasten zu Anregungen im Zahneingriff oder der umliegenden Struktur führen. Die Lastaufteilung auf die Planeten ist dabei abhängig von der individuell für jeden Zahneingriff vorliegenden Steifigkeit. Die Eingriffssteifigkeit ist dabei nicht nur von der Zahnradgeometrie sondern auch von den strukturell angebundenen Komponenten wie Wellen, Gehäuse oder Drehmomentenstützen abhängig. In Windkraftanwendungen werden diese Komponenten häufig sehr steiff ausgeführt, um einen geringen Einfluss auf das Einsatzverhalten auszuüben. In diesem Bericht wird eine Methode vorgestellt, welche eine Strukturoptimierung mit einer Zahnkontaktanalyse für Planetengetriebe kombiniert. Mit Hilfe dieses kombinierten Ansatzes ist es möglich die Masse des Hohlradgehäuses zu reduzieren und gleichzeitig das Einsatzverhalten zu bewerten. Durch die gewichtete Bewertungsfunktion ist es zudem möglich als Auslegungsziel die Lastaufteilung, das Anregungsverhalten, die Masse oder ein ausgewogenes Design festzulegen.

☑ Julian Theling j.theling@wzl.rwth-aachen.de

<sup>1</sup> Laboratory of Machine Tools and Production Engineering (WZL), Campus Boulevard 30, Aachen, Germany

<sup>2</sup> RWTH Aachen, Aachen, Germany

# 1 Introduction and motivation

In 2012, about 32% of the companies in the German mechanical and automotive engineering sector used lightweight construction technologies [1]. The term lightweight construction includes the use of lightweight construction materials such as high-strength steels or fiber composites (lightweight material construction), but also structural



Fig. 1 Percentage of Material Costs (Left) and Advantages of Structural Optimization (Right)

optimization for better utilization of the existing material (lightweight design) and the integration of additional functions in the supporting structure (function-integrated lightweight construction) [2].

The reasons for using lightweight construction technologies depend mainly on the product to be manufactured. The main factors are often the reduction of mass or material costs. Fig. 1 shows on the left the cost types in relation to the total costs of automotive suppliers in Germany. Due to the high level of automation in production, the material accounts for approx. 56% of the total costs incurred. In comparison to this the personnel costs in the high-wage country Germany are approx. 18% of the total costs [3]. Due to the reduction of assigned material and a better material utilization, potential for cost reduction and profit maximization often results. This cost structure is not directly transferable into wind turbine industry, but shows that with a high percentage of automated processes, material costs become more important.

The distribution of the masses at the FVA nacelle gearbox, see Fig. 1, shows that approx. 35% of the gearbox weight is accounted for by the external structural components (housing, ring gear, torque arm) [4]. In comparison, approx. 65% of the mass is accounted for by the rotating, power-transmitting components. Since a large part of the external structural components are casted from ductile cast iron, the material costs are lower compared to the casehardened steel of the gears. Savings are nevertheless possible, since castings in particular are produced close to the final contour and there is no need to remove excess material by machining. A structural optimization and the associated reduction of the mass of the external components leads to lower material demand and also to lower transportation costs. With lower mass, smaller cranes or machines can be used, for example, so that the need for infrastructure in the company and during assembly is lower [5].

Besides mass reduction, the optimization of the deformation behavior is an essential aspect of structural optimization. Planetary gearboxes in particular react sensitively to axis deviations. Since axis deviations do not result in ideal contact of the tooth flanks, excitations can occur, but also a changed load distribution and thus loading of the individual planets. With the help of a structural optimization an optimization of the structure regarding optimal displacement behavior and thus better contact conditions in the gear mesh is possible.

The use of lightweight construction technologies in gear construction is limited in most areas of application to the use of more stressable materials or the reduction of the stress on the gear teeth and thus the reduction of mass through reduced gear widths or center distances [6]. Furthermore, there are approaches to reduce the mass of the gear body, which is subject to less stress [7]. An optimization of the gear environment, especially of planetary gear stages, has hardly been part of scientific investigations so far and could provide further potentials for increasing the power density.

# 2 Structure optimization of planetary gearboxes

The structural optimization tasks are subdivided on the basis of the degrees of freedom in the optimization, since the solution strategy can also be derived from them [8]. In addition to the classification according to the selection of

Content courtesy of Springer Nature, terms of use apply. Rights reserved.

the construction method and the material used, optimization tasks are divided into three further classes. The class with the smallest degree of freedom after the choice of material is the dimensioning. Here the shape of the component is left unchanged and individual geometry parameters are adjusted. In the case of a simple beam, three different parameters (length, width, height) are available for dimensioning, which can be optimized for the load [5].

If the outer contour of the component is also changed, it is called shape optimization. In shape optimization, the outer shape of the component is adapted so that different profiles can exist over the length of the beam. If it is possible to add new structural elements such as holes, cutouts or beams within the optimization, it is called topology optimization. Usually, a higher number of degrees of freedom leads to enhanced solutions. At the same time, the computational effort increases, so that it has to be considered to what extent the effort is tolerable [5].

The most used operating form of planetary gearboxes in wind-industry is the two-shaft operation. This means that one of the three central shafts (planet carrier, sun gear, ring gear) is fixed to the housing and the input or output is realized via the other two shafts. For the input stage, which has to carry the highest torque, gearboxes with a housing fixed ring gear are mainly used in industrial practice. Therefore, this report is limited to this design.

For structural optimization of planetary gears, heavy, constantly loaded components are primarily suitable. In literature, therefore, there are mostly approaches for structural optimization of planet carriers, among other factors mainly because the rotating mass of the carrier can modify the rotational transmission and inertia behavior [9–11]. Due to the fact that the direction of force is always the same, the

planet carrier has a constant point of force application and a constant load zone. The optimization can therefore, theoretically, be carried out with the worst-case load case.

Due to its weight, the connection of the ring gear to its surroundings is another suitable element for structural optimization. The gear housing supports the torque applied to the ring gear and transfers it into the main frame. Torque arms or discrete connection points in form of screwed or welded connections are often chosen for this purpose. In literature, no work for the optimization of the ring gear housings can be found.

In a planetary gearbox the power is split between power paths (planets) due to multiple meshes, so that not only one load acts simultaneously on the ring gear, but the number of loads corresponds to the number of planets, see Fig. 2 (left). In a zero-deviation gear unit, the amount of the input torque which is transmitted by each planet is determined by the combination of all meshing stiffnesses [12]. The load that is transmitted by each individual mesh is also variable over the tooth width, see Fig. 2 (bottom left).

In contrast to the planet carrier, the direction of force and the point of force application in the ring gear changes depending on the position of the carrier, see Fig. 2 (top right). Depending on the position of the planets, a different area of the structure is thus loaded and lies within the force flow. This means that an optimization cannot be carried out for a discrete carrier position, but a multitude of mesh positions over the circumference of the ring gear must be considered.

The connection of the ring gear to be optimized additionally influences the meshing stiffness of the individual ring gear mesh positions, see Fig. 2 (bottom right). Due to the often-used discrete connection to the environment, certain



Fig. 2 Challenges during the Structural Optimization of Planetary Gearbox Housings

areas react more rigidly to the load in the tooth mesh than other areas. As a result, the stiffness of the mesh is different over the circumference. Depending on the position of the planet carrier, this also leads to a variable load distribution, so that the load cases for different housing structures vary and cannot be precalculated.

# 3 Extension of the FE-Stirnradkette in order to simulate planetary gearboxes

In the planetary gearbox, the load applied to the central gears is divided between the individual planets. How much load is transmitted via a power path depends on the meshing conditions of all gear meshes of a planetary gear unit. In order to be able to predict the load distribution and thus the locally correct load on the housing structure caused by the ring gear meshes, the FE-based tooth contact analysis STIRAK is extended in three steps. First the simultaneous loading of the central gears with all planet meshes is implemented, then the single gear meshes are coupled in a spring model. In a further step, the housing structure is connected to the ring gear so that the surrounding stiffnesses can be taken into account in the tooth contact analysis. The work presented here is part of the IGF project 19328N/1 [13], which is also known under the FVA project number FVA 377 II [14].

# 3.1 Extension of the calculation of influence coefficients

The FE-Stirnradkette is an FE-based tooth contact analysis. This means that the tooth contact stiffness is not determined by an analytical method, but by an upfront calculation of influence numbers in an FE model. For this purpose, loadfree contact points on the line of contact are precalculated for each contact position and individually loaded with a unit load in the FE model. The displacements of the loaded contact point are then evaluated, as well as the displacements of all unloaded contact points of the same mesh position. By the defined force and the calculated path of deformation of each contact point, a stiffness or its reciprocal value, the influence number, can be determined. The final result of the influence number calculation is a matrix, which describes the deformation of n points due to a single force at each of these *n* points, see Fig. 3. Together with the contact distances and the overall load, the tooth mesh is solved numerically on the basis of a spring model. The deformation of the springs and the force transmitted by each spring can be used to determine the load in the gear mesh.

Up to now, the FE Stirnradkette calculates the influence numbers for each mesh individually, see Fig. 3 top right. If there is more than one mesh on a gear, a single mesh calculation is performed for each mesh. In this way, influences from the mesh with the first planet cannot be considered in the mesh with the second planet. This loss of information is eliminated by determining the influence numbers for multiple meshes, see Fig. 3 bottom right. This means that the deformation of the contact points is evaluated in all meshes. In this way, the deformation of a contact point in contact with the second planet is also determined based on the load at a contact point with the first planet.

This results in changed total deformations of the system. The analysis of the deformation of a sun wheel, which is in mesh with three planets and is exclusively bound to one face, shows the influence of the multiple mesh, see Fig. 3



Fig. 3 Extension of the Influence Coefficient Calculation

Deringer

Content courtesy of Springer Nature, terms of use apply. Rights reserved.

right. When loaded with the force from a single mesh, the sun gear is displaced by the existing radial force. With multiple meshes, the radial forces of the individual meshes balance each other out to a large extent and the wheel remains centered. Instead, a torsional deformation occurs across the width of the gear, which is greater than in single mesh conditions. This increased torsional deformation and the reduction of the displacement corresponds to the real deformation and load behavior, so that the calculation quality of planetary gearboxes increases.

#### 3.2 Extension of the spring model

In addition to the correct representation of the stiffnesses and deformations of the individual gears, the load distribution in the overall system must also be known in order to correctly determine the loads in the planetary gearbox. For this purpose, the spring model, which up to now has determined the load distribution to the individual teeth and contact points, is extended, see Fig. 4.

For this purpose, it is necessary that the solution of the spring model is holistic, i.e. that it covers all meshes simultaneously, since all meshes influence each other. For this



Fig. 4 Extension of the Spring Model



Fig. 5 Connection of the Housing Structure to the Ring Gear



purpose, the planetary gearbox is modelled as a series and parallel connection of springs. Each single gear mesh is modeled as a parallel connection of the individual springs of contact points.

For the solution of the overall system, the submodels of planetary (series connection) and central gears (parallel connection) are combined to an overall model. For the solution of the overall model, an equation system is constructed. By using force and torque equilibrium and other boundary conditions, the model can be solved unambiguously.

# 3.3 Connection between the housing structure and the ring gear

The stiffness of the gear mesh is determined by the support of the forces applied in the environment. Depending on the local flexibility of the housing structure, the gear mesh deforms differently. As already shown in the previous chapter, the meshing stiffness influences the load distribution in the planetary gearbox and thus also the load of the individual meshes.

The coupling of the ring gear with the surrounding housing structure is therefore carried out [14]. For this purpose, the housing, including the connection to the environment, is modeled and meshed in an external software, see Fig. 5.

Within the program, the ring gear and the housing are aligned with each other and bonded together at the contact nodes so that radial and tangential forces can be transmitted. Finally, the calculation of the influence numbers is done according to the method described in Sect. 3.2. The stiffnesses of the housing structure are thus included in the influence numbers and can be used to solve the gear mesh. The method described enables the solution of the model for one transverse contact pitch in the planetary gearbox. The following pitch, on the other hand, may have different stiffness ratios, since other teeth on the ring gear are loaded and the direction of force changes. In order to be able to take these conditions into account, the method is extended in a way that several pitches can be rolled off one after the other, Fig. 5right. The individual results are combined so that the effects resulting from the stiffness of the ring gear housing structure, which varies over the circumference, can be regarded.

# 4 Method in order to optimize the housing structure in combination with the tooth contact analysis

By taking into account the housing structure in the tooth contact analysis it is possible to directly regard the influence of a housing design on the operational behavior. This advantage can be used to design the housing in a way, that it has a positive influence on the gear mesh. Therefore, a method is developed which optimizes the shape of the housing by evaluating the resulting operational behavior and the mass. Before an optimization can be carried out, the component to be optimized must be parameterized, see Fig. 6. As an example a generic test gearbox is created which is based on the design of the FVA nacelle [4]. The number of teeth at the ring and sun gear are adapted in a way, that a symmetrical meshing sequence with a theoretically ideal load distribution is achieved. The housing is connected to the main frame by two torque arms.



Fig. 6 Overview of the Optimization Method

Deringer

Content courtesy of Springer Nature, terms of use apply. Rights reserved.



Fig. 7 Results of the Optimization

The inner diameter of the ring gear housing, where the ring gear itself is mounted as well as the planet carrier bearing and the connection to the main frame are not optimized because they are fixed interfaces to the environment. In the example shown here, 40 points over the circumference describe the outer contour of the ring gear housing. Because of the symmetrical design only ten points are optimized for one quadrant and mirrored into the other three quadrants. The individual points are automatically connected via splines. To reduce the number of optimization parameters, the points are assigned a fixed angular position. The distribution over the circumference is done in even angular steps. The parameter for each point released for optimization is the radius  $r_{y}$ . Thus, the rim thickness of the ring gear housing can be optimized locally over the circumference. The parameterization of the torque arm is done via three points. The points are also connected to each other and to the interface to the main frame via splines.

Fig. 6 shows the procedure for the optimization of the housing structure. In the first step values are assigned to the parameters enabled for optimization. This is done using a genetic or evolutionary optimization algorithm in MAT-LAB [15]. Here, processes from the theory of evolution are used to adapt the housing structure to the given conditions. Similar to nature, the optimization is performed over several generations. A generation consists of a certain number of individuals, which differ from each other in their parameterization. After the parameters have been defined, the geometry is automatically generated for each individual and meshed to an FE model in ABAQUS.

The extended FE-Stirnradkette uses for the analysis of the generated geometry, then the FE model as well as the information of the bondage and the connection of the ring gear. The example gearbox is symmetrical, has a ring gear with  $z_{Ring} = -90$  and three planets are installed with an angular offset of = 120°. After a third of a full rotation of the planet carrier, the second planet is at the starting position of the first planet, so that the sequence is repeated. Therefore, a calculation of thirty pitches is sufficient.

After the analysis, the results are weighted and evaluated. As evaluation parameters, characteristic values for the loading and excitation as well as the weight of the structure are determined. In the example shown here, the loading is evaluated by the force fluctuation  $\Delta F_{bt}$ , which, because of the symmetrical meshing sequence, depicts the load distribution to the individual power paths. The value is derived for a complete revolution of the planet carrier. The transmission error between input and output, i.e. between the planet carrier and the sun, is determined as a parameter to describe the excitation behavior. The quantities that are included in the evaluation are the gear mesh orders. The mass of the housing structure is determined by the volume of the FE-model and the density of the material. All single values are normalized with the results of the original variant due to the different orders of magnitude, so that a value smaller than one represents an improvement. Mass, excitation behavior and load sharing are all rated equally in order to achieve a balanced design.

#### 5 Results of the optimization

Within the optimization process more than 300 variants were generated and their operational behavior was derived and rated. The resulting geometry with the best overall rating is shown in Fig. 7. The reference variant is based on

the original design of the FVA nacelle and uses the identical rim thickness of  $Rk = 5.5 \cdot m_n = 100 \text{ mm}$  with an outer diameter of d=960 mm [4].

It can be observed, that the rim of the ring gear housing is not equally thick in comparison to the reference, see Fig. 7 lower left picture. Especially the area around the torque arm and the torque arm itself differ from each other. The torque arm is designed thicker where it connects to the housing, but material is removed directly at the coupling to the main frame. The ring gear rim of the optimized variant is thinner in these locations, whereas the thickness at the  $0^{\circ}$  and  $180^{\circ}$ position increases. The optimized variant leads to a mass reduction of 113 kg, which is 3.8% of the reference value.

In the upper middle diagram of Fig. 7 the transmission error of the reference variant is shown. The values are presented in order domain whereas the 90th order represents the first gear mesh order 1.O<sub>G</sub>. The transmission error between carrier and sun of the reference variant shows the largest excitation at the first gear mesh order with  $TE_{1.0G}$  = 3.73 µm. Some smaller excitations are visible in lower orders, which result out of the varying ring gear stiffness over the circumference. One peak is visible at the 12th order. The 12th order is excited because three planets are mounted and the two torque arms provide the maximum stiffnesses at four locations. At these locations the gear mesh force is directed directly onto the coupling to the main frame or in the exact opposite position. This phenomenon can also be observed when looking at the gear force variation of one planet, see Fig. 7 lower middle diagram. The load, which is transmitted by one planet, is variable over the circumference. Because of an working pressure angle of  $\alpha_{wt} = 20^\circ$  at  $\tau = 160^\circ$  and  $\tau = 340^\circ$  the least load is carried because the gear mesh force is directed on the least stiff 180° or 360° position. The highest loads appear at  $\tau = 110^{\circ}$  and  $\tau = 290^{\circ}$ .

In order to compare the optimized and the reference geometry, the difference in the transmission error order spectrum and force variation is derived, see Fig. 7 right diagrams. Values lower than zero show a reduction, values higher than zero an increase of the transmission error or force.

It can be observed, that the optimized variant shows a reduction of the transmission error in the first gear mesh order with  $\Delta TE = -0.13 \,\mu$ m. Higher orders are not significantly influenced whereas lower orders are both, increased and reduced. The 6th carrier rotational order and their multiples show higher amplitudes than in the reference. All other carrier rotational orders can be reduced up to the 50th order.

Due to the optimized design, the force variation at one planet is influenced. In areas where the planet carries the most load ( $\tau = 110^{\circ}$  and  $\tau = 290^{\circ}$ ) the load can be reduced. This decreases the peak load and optimizes the durability of

the gear stage. In other areas where less load is transmitted by one planet, the values increase slightly.

All in all the method is capable to optimize the operational behavior of planetary gear stages and reduce the mass of the system. Due to the high rim thickness of the reference design the operational behavior is already nearly constant over the circumference. Therefore, improvements regarding the transmission error and the force excitation are comparably small. Nevertheless, improvements can be observed as well as a mass reduction of  $\Delta m = 3.8\%$  or 113 kg. The saved material has not to be molten, transported or purchased and can therefore, reduce costs for the manufacturer and customer.

#### 6 Summary and outlook

This report deals with the shape optimization of the housing structure of planetary gearboxes under consideration of the interactions between the tooth mesh and its surroundings. In addition to material, parameter and topology optimization, shape optimization is a discipline of structural optimization, which is characterized by the fact that the outer contour of components is optimized in shape and dimension. A frequent goal of structural optimization is to save material and thus reduce material costs and component weight.

In transmission technology, the use of lightweight design often comes in the form of lightweight material design. In particular, the highly stressed gear teeth are made of a material that can withstand higher loads in order to reduce the dimensions of the gear box. In addition to the gear teeth, a significant part of the mass of the FVA nacelle gear box, approximately 35%, is taken up by the outer structural components.

A method is presented which allows to combine the tooth contact analysis and a structural optimization approach. Therefore, the tooth contact analysis FE-Stirnradkette is extended in order to simulate planetary gear stages and regard the influence of the housing structure of ring gears. The structural optimization of the ring gear housing is carried out by means of a genetic algorithm, which optimizes the outer contour. This leads to variable rim thicknesses over the circumference. The optimized geometry is stiffened in areas where the reference design is the most flexible and stiffness is reduced in the areas around the toque arms.

All in all, the optimized geometry is capable to reduce the excitation and improve the load sharing behavior while saving mass. In this particular case the transmission error of the first gear mesh order can be reduced by  $\Delta TE = -0.13 \,\mu m$ and the peak load can be reduced by  $F_{bt} = 1.1 \,kN$ . The optimized geometry saves 113 kg of mass in comparison to the reference. The improvements in the operational behavior are small, because the reference design was designed very stiff. Modern gearboxes often use more flexible ring gears in order to improve the load sharing behavior. The effect of the optimization on a less stiff ring gear has to be tested in the future. Also the load carrying capacity as well as dynamic influences should be taken into account.

Acknowledgements The authors gratefully acknowledge financial support by the Research Association for Power Transmission Engineering (FVA) [FVA 377 II] for the achievement of the project results.





The authors gratefully acknowledge the Federal Ministry for Economic Affairs and Energy for providing the financial support for the project [19328 N/1] based on a resolution of the German Parliament

Funding Open Access funding enabled and organized by Projekt DEAL.

**Open Access** This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4. 0/.

#### References

- Leichtbau BW Gmbh (2014) Leichtbau Trends und Zukunftsmärkte: und deren Bedeutung für Baden-Württemberg. https://www.ipa.fraunhofer.de/content/dam/ipa/de/documents/ Publikationen/Studien/Studie\_LeichtbauBW\_Trends\_ Zukunftsmaerkte.pdf. Accessed: 20 March 2020
- Bundesministerium für Wirtschaft und Energie (2020) Schlüsseltechnologien: Leichtbau. https://www.bmwi.de/Redaktion/DE/ Dossier/leichtbau.html. Accessed 31 Mar 2020
- Commerzbank (2013) Verteilung der Kostenarten an den Gesamtkosten der Automobilzulieferer in Deutschland im Jahr 2013. https://de.statista.com/statistik/daten/studie/314397/umfrage/ kostenarten-anteil-gesamtkostenkosten-autozulieferer/. Accessed 31 Mar 2020
- Bosse D (2019) Abschlussbericht FVA Gondel Belastungen an Antriebskomponenten von Windenergieanlagen: Forschungsvorhaben Nr. 730. FVA Heft 2019, pp 1–136
- Schumacher A (2013) Optimierung mechanischer Strukturen: Grundlagen und industrielle Anwendungen, 2nd edn. Springer Vieweg, Berlin
- 6. ISO (2006) Calculation of load capacity of spur and helical gears. Calculation of surface durability (pitting)(6336 Part 2)
- Mandt D (2006) Eigenschaften und Einsatzverhalten von leichten und dämpfenden Werkstoffverbund-Zahnrädern. Diss., RWTH Aachen University
- Schmit LA Jr., Mallet RH (1963) Structural synthesis and design parameter hierarchy. J Struct Div 1963:269–300
- Dong W, Xing Y, Moan T (2012) Time domain modeling and analysis of dynamic gear contact force in a wind turbine gearbox with respect to fatigue assessment. Energies 5:4350–4371. https://doi. org/10.3390/en5114350
- Yi P, Dong L, Shi T (2014) Multi-objective genetic algorithms based structural optimization and experimental investigation of the planet carrier in wind turbine gearbox. Front Mech Eng 9:354–367. https://doi.org/10.1007/s11465-014-0319-5
- Xing Y, Moan T (2013) Multi-body modelling and analysis of a planet carrier in a wind turbine gearbox. Wind Energy 16:1067–1089. https://doi.org/10.1002/we.1540
- Heider MK (2012) Schwingungsverhalten von Zahnradgetrieben: Beurteilung und Optimierung des Schwingungsverhaltens von Stirnrad und Planetengetrieben. Diss., TU München
- Brecher C, Brimmers J, Theling J (2020) Zwischenbericht zu IGF Vorhaben 19328N/1: Berechnungsmethode zur Berücksichtigung der gegenseitigen Beeinflussung von Einzelzahneingriffen im Mehrfacheingriff von Planetengetrieben
- 14. Brecher C, Brimmers J, Theling J (2020) Sachstandsbericht zu FVA Vorhaben 377 II: Berechnungsmethode zur Berücksichtigung der gegenseitigen Beeinflussung von Einzel-zahneingriffen im Mehrfacheingriff von Planetengetrieben
- MathWorks (2020) MATLAB documentation. https://de.mathworks. com. Accessed 5 Feb 2021

# Terms and Conditions

Springer Nature journal content, brought to you courtesy of Springer Nature Customer Service Center GmbH ("Springer Nature").

Springer Nature supports a reasonable amount of sharing of research papers by authors, subscribers and authorised users ("Users"), for small-scale personal, non-commercial use provided that all copyright, trade and service marks and other proprietary notices are maintained. By accessing, sharing, receiving or otherwise using the Springer Nature journal content you agree to these terms of use ("Terms"). For these purposes, Springer Nature considers academic use (by researchers and students) to be non-commercial.

These Terms are supplementary and will apply in addition to any applicable website terms and conditions, a relevant site licence or a personal subscription. These Terms will prevail over any conflict or ambiguity with regards to the relevant terms, a site licence or a personal subscription (to the extent of the conflict or ambiguity only). For Creative Commons-licensed articles, the terms of the Creative Commons license used will apply.

We collect and use personal data to provide access to the Springer Nature journal content. We may also use these personal data internally within ResearchGate and Springer Nature and as agreed share it, in an anonymised way, for purposes of tracking, analysis and reporting. We will not otherwise disclose your personal data outside the ResearchGate or the Springer Nature group of companies unless we have your permission as detailed in the Privacy Policy.

While Users may use the Springer Nature journal content for small scale, personal non-commercial use, it is important to note that Users may not:

1. use such content for the purpose of providing other users with access on a regular or large scale basis or as a means to circumvent access control;

- 2. use such content where to do so would be considered a criminal or statutory offence in any jurisdiction, or gives rise to civil liability, or is otherwise unlawful:
- 3. falsely or misleadingly imply or suggest endorsement, approval, sponsorship, or association unless explicitly agreed to by Springer Nature in writing;
- 4. use bots or other automated methods to access the content or redirect messages
- 5. override any security feature or exclusionary protocol; or
- 6. share the content in order to create substitute for Springer Nature products or services or a systematic database of Springer Nature journal content.

In line with the restriction against commercial use, Springer Nature does not permit the creation of a product or service that creates revenue, royalties, rent or income from our content or its inclusion as part of a paid for service or for other commercial gain. Springer Nature journal content cannot be used for inter-library loans and librarians may not upload Springer Nature journal content on a large scale into their, or any other, institutional repository.

These terms of use are reviewed regularly and may be amended at any time. Springer Nature is not obligated to publish any information or content on this website and may remove it or features or functionality at our sole discretion, at any time with or without notice. Springer Nature may revoke this licence to you at any time and remove access to any copies of the Springer Nature journal content which have been saved.

To the fullest extent permitted by law, Springer Nature makes no warranties, representations or guarantees to Users, either express or implied with respect to the Springer nature journal content and all parties disclaim and waive any implied warranties or warranties imposed by law, including merchantability or fitness for any particular purpose.

Please note that these rights do not automatically extend to content, data or other material published by Springer Nature that may be licensed from third parties.

If you would like to use or distribute our Springer Nature journal content to a wider audience or on a regular basis or in any other manner not expressly permitted by these Terms, please contact Springer Nature at

onlineservice@springernature.com