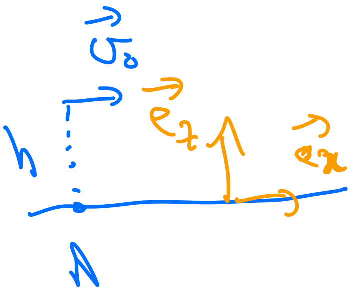
Exo 1



1) Objet en chute libre : axe (Oz)

$$\cancel{m}\ddot{z} = -\cancel{m}g$$

$$\dot{z} = -gt + \cancel{v_{0z}}$$ pas de vitesse initiale sur (Oz)

$$z(t) = -\frac{1}{2}gt^2 + z_0 \qquad z(0) = h$$

$$\boxed{z(t) = -\frac{1}{2}gt^2 + h}$$

$$z(\tau) = 0 = -\frac{1}{2}g\tau^2 + h \qquad \tau^2 = \frac{2h}{g} \qquad \boxed{\tau = \sqrt{\frac{2h}{g}}}$$

A.N : $\tau = 10{,}1\ s$

2) $\boxed{d = v_0 \times \tau}$ A.N : $250\ km/h \xrightarrow{/3{,}6} 69{,}4\ m/s$

$d = 701\ m$

3) distance d ↓ si vitesse $\vec{v}_0$ selon $\vec{e}_x$ pour le colis.

4) 

Chute libre encore

$$\begin{cases} m\ddot{z} = -mg \\ \ddot{x} = 0 \end{cases} \qquad \begin{cases} \dot{z} = -gt - v_0 \sin\beta \\ \dot{x} = v_0 \cos\beta \end{cases}$$

$$\begin{cases} z(t) = -\frac{1}{2}g t^2 - v_0 \sin\beta\, t + h \\ x(t) = v_0 \cos\beta\, t \end{cases}$$

$$z(\tau') = 0 \Leftrightarrow -\frac{1}{2}g\tau'^2 - v_0\sin\beta\,\tau' + h = 0$$

$$\Delta = (v_0 \sin\beta)^2 + 4 \times \frac{1}{2} g h = (v_0\sin\beta)^2 + 2gh$$

$$\boxed{\tau' = \frac{+v_0 \sin\beta \pm \sqrt{\Delta}}{-g} = \frac{-v_0\sin\beta}{g} + \sqrt{\left(\frac{v_0\sin\beta}{g}\right)^2 + \frac{2h}{g}}}$$

seule solution acceptable ($\tau' > 0$)

A.N: $\boxed{\tau' = 7{,}2\,s}$

$\boxed{d' = v_0 \cos\beta \times \tau'}$ A.N: $\boxed{d' = 431\,m}$

5) on veut $d' = 100\,m \rightarrow \tau' = \frac{d'}{v_0\cos\beta} = 1{,}66\,s$

avec $\tau' = \frac{-v_0\sin\beta + \sqrt{\Delta}}{g} = \frac{-v_0\sin\beta}{g} + \sqrt{\left(\frac{v_0\sin\beta}{g}\right)^2 + \frac{2h'}{g}}$

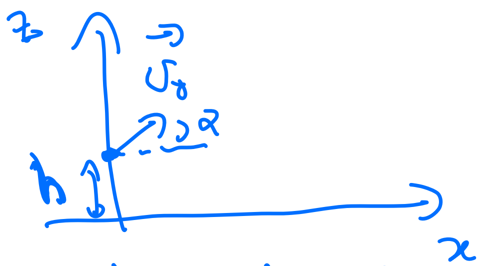
$$\tau' + \frac{v_0 \sin\beta}{g} = \sqrt{\left(\frac{v_0 \sin\beta}{g}\right)^2 + \frac{2h'}{g}}$$

$$\left(\tau' + \frac{v_0 \sin\beta}{g}\right)^2 = \left(\frac{v_0 \sin\beta}{g}\right)^2 + \frac{2h'}{g}$$

$$\boxed{h' = \frac{g}{2}\left[\left(\tau' + \frac{v_0 \sin\beta}{g}\right)^2 - \left(\frac{v_0 \sin\beta}{g}\right)^2\right]}$$

A.N : $h' = 71\ m$

Exo 2 :



1) Chute libre :
(seul le poids compte)

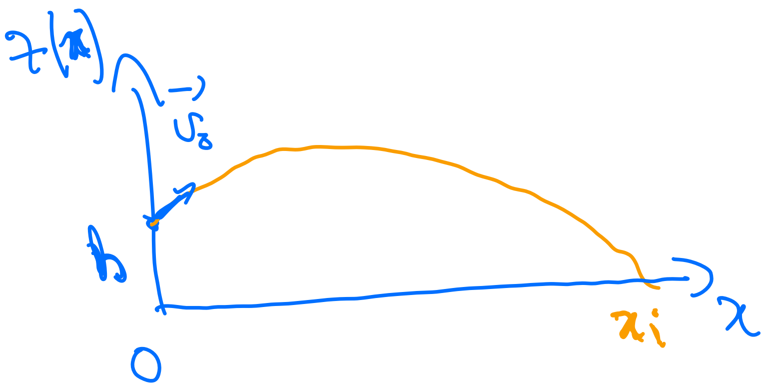
$$\begin{cases} m\ddot{x} = 0 \\ m\ddot{z} = -mg \end{cases} \qquad \begin{cases} \dot{x} = v_0 \cos\alpha \\ \dot{z} = -gt + v_0 \sin\alpha \end{cases}$$

$$\boxed{\begin{cases} x(t) = v_0 \cos\alpha\, t + \cancel{x_0} \\ z(t) = -\frac{1}{2} g t^2 + v_0 \sin\alpha\, t + h \end{cases}}$$

2) Trajectoire $z(x)$

$$t = \frac{x}{v_0\cos\alpha} \Rightarrow z(x) = -\frac{1}{2} g \frac{x^2}{(v_0\cos\alpha)^2} + v_0\sin\alpha \frac{x}{v_0\cos\alpha} + h$$

$$\boxed{z(x) = -\frac{1}{2(v_0\cos\alpha)^2}\, g\, x^2 + \tan\alpha\, x + h}$$



3) $z(x_i) = 0$

$$0 = \frac{-g}{2(v_0\cos\alpha)^2} x_i^2 + \tan\alpha\, x_i + h$$

$$D = \tan^2\alpha + \cancel{4}\,2\,\frac{g}{\cancel{2}\,(v_0\cos\alpha)^2} \times h$$

$$\boxed{x_i = \frac{-\tan\alpha \pm \sqrt{D}}{\cancel{2}\times\frac{-g}{\cancel{2}(v_0\cos\alpha)^2}} = \frac{(v_0\cos\alpha)^2}{g}\left[\tan\alpha \mp \sqrt{D}\right]}$$

solut° acceptable physiquement.

A.N: $\boxed{x_i = 13{,}7\ m}$

4) $t_i = \dfrac{x_i}{v_0 \cos\alpha}$ A.N: $t_i = 1{,}3\ s$

5) z_{max} ?

$\dfrac{dz}{dx} = 0$ $\quad -\dfrac{g\, x_{max}}{(v_0 \cos\alpha)^2} + \tan\alpha = 0$

$$x_{max} = \frac{\tan\alpha\, v_0^2 (\cos\alpha)^2}{g}$$

$$\boxed{x_{max} = \frac{v_0^2}{g} \sin\alpha \cos\alpha}$$

A.N: $6{,}35\ m$

$z_{max} = z(x_{max})$

$$= -\frac{1}{2(v_0 \cos\alpha)^2}\, g \left(\frac{v_0^2}{g} \sin\alpha\cos\alpha \right)^2 + \tan\alpha\, \frac{v_0^2}{g} \sin\alpha\cos\alpha + h$$

$$= -\frac{1}{2g} v_0^2 \sin^2\alpha + \frac{v_0^2}{g} \sin^2\alpha + h$$

$$\boxed{z_{max} = \frac{1}{2g} v_0^2 \sin^2\alpha + h}$$

A.N: $z_{max} = 7{,}40\ m$

Exo 3

1) $(M+m)\ddot{z} = -(M+m)g + \alpha \dot{z}^2$

$$\ddot{z} - \frac{\alpha}{(M+m)}\dot{z}^2 + g = 0$$

2) vitesse limite : $\ddot{z} = 0 \Rightarrow -\frac{\alpha v_l^2}{(M+m)} + g = 0$

$$v_l^2 = \frac{M+m}{\alpha} g$$

$$\boxed{v_l = \sqrt{\left(\frac{M+m}{\alpha}\right) g}}$$

A.N : $v_l = \sqrt{\frac{100}{20} \times 10}$

$v_l = \sqrt{50} \sim 7{,}1 \, m/s$

$\sim 25{,}5 \, km/h$

3) chute libre depuis hauteur h

$m\ddot{z} = -mg \qquad \ddot{z} = -g$

$\dot{z} = -gt + \cancel{v_0}$

$z = -\frac{1}{2} g t^2 + h$

$$\dot z(\tau) = -v_L \Rightarrow \tau = \frac{v_L}{g}$$

$$z(\tau) = 0 \Leftrightarrow -\frac{1}{2} g \tau^2 + h = 0$$

$$\boxed{h = \frac{1}{2} g \tau^2 = \frac{1}{2} g \frac{v_L^2}{g^2} = \frac{1}{2} \frac{v_L^2}{g}}$$

A.N: $\boxed{h = 2{,}5 \text{ m}}$

Exo 4

Système masse ; dans le référentiel terrestre supposé galiléen.

Bilan des forces : poids, force de rappel + frottement fluide

1) $$m\ddot z = -mg - k(l - l_0) - 6\pi\eta r \dot z$$

avec l'élongation $z = (l - l_0)$

$$\boxed{\ddot z + \frac{6\pi\eta r}{m}\dot z + \frac{k}{m} z = -g}$$

Etude du régime transitoire : solut° de l'équation homogène.

$$\ddot{z} + \frac{6\pi\eta r}{m}\dot{z} + \frac{k}{m}z = 0$$

2) polynôme caractéristique : $p^2 + \frac{6\pi\eta r}{m}p + \frac{k}{m} = 0$

$$\Delta = \left(\frac{6\pi\eta r}{m}\right)^2 - 4\frac{k}{m}$$

pseudo période pour le régime pseudo-périodique $\Delta < 0$

$$p = -\frac{6\pi\eta r}{2m} \pm \frac{j}{2}\sqrt{-\Delta}$$

$$z(t) = e^{-\frac{6\pi\eta r}{2m}t}\left[A\cos(\Omega t) + B\sin(\Omega t)\right]$$

avec $\Omega = \frac{\sqrt{-\Delta}}{2}$

donc la pseudo période $T = \frac{2\pi}{\Omega} = \frac{4\pi}{\sqrt{-\Delta}}$

$$T = \frac{4\pi}{\sqrt{-\left(\frac{6\pi\eta r}{m}\right)^2 + \frac{4k}{m}}}$$

d'l'ans l'udes $T_0 = \frac{2\pi}{\omega_0}$ avec $\omega_0 = \sqrt{\frac{k}{m}}$

$$\frac{1}{T^2} = \frac{1}{T_0^2} - \frac{9\eta^2 r^2}{4m^2} \Rightarrow \boxed{\eta = \frac{2m}{3r}\sqrt{\frac{1}{T_0^2} - \frac{1}{T^2}}}$$

$\Rightarrow$