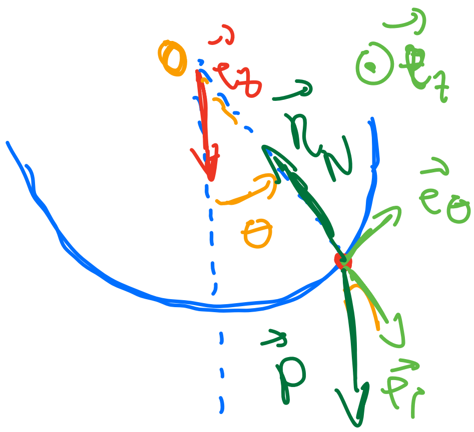
Exo 1



1) Bilan des forces.

$\vec{P}$ et $\vec{R}_N$ (pas de frottement)

– $\vec{R}_N$ toujours colinéaire à $\vec{OM}$

$\Rightarrow \vec{\Gamma}_{O,\vec{R}_N} = \vec{OM} \wedge \vec{R}_N = \vec{0}$

– $\vec{P}$: $\vec{\Gamma}_{O,\vec{P}} = \vec{OM} \wedge \vec{P} = \begin{vmatrix} R \\ 0 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} mg\cos\theta \\ -mg\sin\theta \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ -mgR\sin\theta \end{vmatrix}$

expression dans la base $(\vec{e}_r, \vec{e}_\theta, \vec{e}_z)$

2) $\vec{L}_O = \vec{OM} \wedge \vec{p}$

$= R \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} \wedge m \begin{vmatrix} 0 \\ R\dot{\theta} \\ 0 \end{vmatrix}$

$$\boxed{\vec{L}_O = mR^2\dot{\theta}\,\vec{e}_z}$$

3) Théo du moment cinétique : $\frac{d\vec{L}_O}{dt} = mR^2\ddot{\theta}\,\vec{e}_z = -mgR\sin\theta\,\vec{e}_z$

$$\Rightarrow \boxed{\ddot{\theta} + \frac{g}{R}\sin\theta = 0}$$

4) approximation des petits angles

$\theta \ll 1 \Rightarrow$ où $\theta \sim \theta$

$$\boxed{\ddot{\theta} + \frac{g}{R}\theta = 0} \Leftrightarrow \ddot{\theta} + \omega_0^2\theta = 0$$

↳ Eq diff du 1er ordre à coeff constant sans 2nd membre :

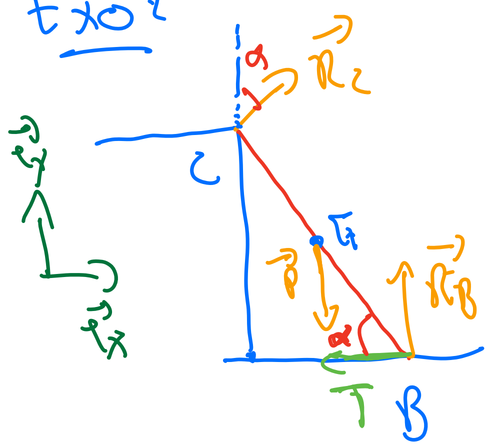
pôly caractéristique : $r^2 + \omega_0^2 = 0 \Rightarrow r = \pm i\omega_0$

solution $\theta(t) = A\cos(\omega_0 t + \varphi)$

mouvement d'oscillation de pulsation $\omega_0 = \sqrt{\frac{g}{R}}$

$$\boxed{T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{R}{g}}}$$

Exo 2



1) Bilan des forces : Poids, les réactions du sol $\vec{R_B}$ et du mur $\vec{R_C}$ + tension du fil attaché en B

2) PFD : à l'équilibre :

$$\begin{cases} \text{Sur } (Ox) : & -T + R_C \sin\alpha = 0 \\ \text{Sur } (Oy) : & -mg + R_B + R_C\cos\alpha = 0 \end{cases}$$

Pb : 3 inconnues

Théorème du moment cinétique : appliqué en B

(2 forces s'appliquent au point B)

* $\vec{\tau}_{B,\vec{R}_B} = \vec{BB} \wedge \vec{R}_B = \vec{0}$

* $\vec{\tau}_{B,\vec{T}} = \vec{BB} \wedge \vec{T} = \vec{0}$

* $\vec{\tau}_{B,\vec{P}} = \vec{BG} \wedge \vec{P} = \frac{L}{2}\begin{vmatrix} -\cos\alpha \\ \sin\alpha \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ -mg \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ \frac{L}{2}\cos(\alpha) mg \end{vmatrix}$

$$\boxed{\vec{\tau}_{B,\vec{P}} = \frac{L}{2} mg \cos(\alpha)\, \vec{e}_z}$$

* $\vec{\tau}_{B,\vec{R}_c} = \vec{BC} \wedge \vec{R}_c = L\begin{vmatrix} -\cos\alpha \\ \sin\alpha \\ 0 \end{vmatrix} \wedge \begin{vmatrix} R_c \sin\alpha \\ R_c \cos\alpha \\ 0 \end{vmatrix} = -LR_c(\cos^2\alpha + \sin^2\alpha)\, \vec{e}_z$

$$\boxed{\vec{\tau}_{B,\vec{R}_c} = -LR_c\, \vec{e}_z}$$

à l'équilibre $\vec{L}_B = \vec{0}$

$\frac{d\vec{L}_B}{dt} = \vec{0} \Rightarrow \frac{L}{2} mg \cos\alpha - LR_c = 0$

$$\boxed{R_c = \frac{mg}{2}\cos\alpha}$$

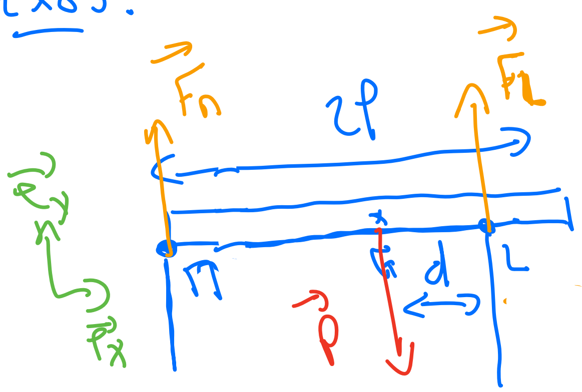
$$\boxed{\vec{R_c} = \frac{mg}{2}\cos\alpha \begin{vmatrix} \sin\alpha \\ \cos\alpha \end{vmatrix}}$$

3) $T = R_c \sin\alpha$ d'après le PFD

$$T = \frac{mg}{2}\cos\alpha\sin\alpha \qquad \boxed{\vec{T} = -\frac{mg}{2}\cos\alpha\sin\alpha\,\vec{e_x}}$$

$$R_B = +mg - R_c\cos\alpha$$
$$= mg\left(1 - \frac{1}{2}\cos^2\alpha\right) \qquad \boxed{\vec{R_B} = mg\left(1 - \frac{1}{2}\cos^2\alpha\right)\vec{e_y}}$$

Exo3 :



1) PFD à l'équilibre

$$F_n + F_L = mg$$

problème 2 inconnues.

Applicat° du Théo du moment cinétique.

en Π

$* \ \vec{\Gamma}_{\Pi,\vec{F_n}} = \vec{0}$

$* \ \vec{\Gamma}_{\Pi,\vec{P}} = \vec{\Pi G} \wedge \vec{P}$

$$\vec{nG}\begin{vmatrix} \ell \\ 0 \\ 0 \end{vmatrix} \quad \vec{P}\begin{vmatrix} 0 \\ -mg \\ 0 \end{vmatrix} \quad \vec{\tau}_{n,\vec{P}} = \begin{vmatrix} \ell \\ 0 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ -mg \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ -mg\ell \end{vmatrix}$$

$$\boxed{\vec{\tau}_{n,\vec{P}} = -mg\ell\,\vec{e_z}}$$

$$* \ \vec{\tau}_{n,\vec{F_L}} = \vec{nL} \wedge \vec{F_L}$$

$$= \begin{vmatrix} \ell+d \\ 0 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ F_L \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ (\ell+d)F_L \end{vmatrix}$$

$$\boxed{\vec{\tau}_{n,\vec{F_L}} = (\ell+d)F_L\,\vec{e_z}}$$

$\frac{d\vec{L_n}}{dt} = \vec{0}$ car à l'équilibre $-mg\ell + (\ell+d)F_L = 0$

$$\boxed{F_L = mg\left(\frac{\ell}{\ell+d}\right)}$$

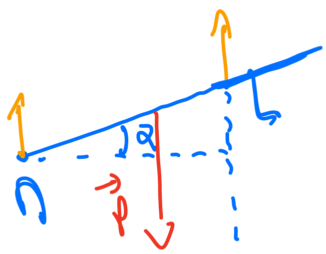
$$F_n = +mg - mg\frac{\ell}{(\ell+d)} = mg\left(\frac{\ell+d}{\ell+d} - \frac{\ell}{\ell+d}\right)$$

$$\boxed{F_n = \frac{d}{\ell + d}\, mg}$$

A.N: $F_L = mg\left(\frac{2}{3{,}4}\right) \sim 0{,}59\, mg$

$F_n = mg\left(\frac{1{,}4}{3{,}4}\right) \sim 0{,}41\, mg$

2)



$$\vec{nF_1} = \begin{vmatrix} \ell\cos\alpha \\ \ell\sin\alpha \\ 0 \end{vmatrix} \qquad \vec{nL} = \begin{vmatrix} (\ell+d)\cos\alpha \\ (\ell+d)\sin\alpha \\ 0 \end{vmatrix}$$

$$\vec{\sigma}_{n,\vec{P}} = \begin{vmatrix} \ell\cos\alpha \\ \ell\sin\alpha \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ -mg \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ -mg\,\ell\cos\alpha \end{vmatrix}$$

$$\vec{\sigma}_{n,\vec{F_L}} = \begin{vmatrix} (\ell+d)\cos\alpha \\ (\ell+d)\sin\alpha \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ F_L \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ (\ell+d)\cos\alpha\, F_L \end{vmatrix}$$

$$-mg\ell\cos\alpha + (\ell+d)\cos\alpha\, F_L = 0$$

l'angle ne change rien !

$$\boxed{F_L = mg\frac{\ell}{(\ell+d)}}$$