

TD 1

Two tanks system (phase plane)

Objective : Study a nonlinear second order system with the phase plane and run simulations.

Consider a hydraulic system with two tanks. The first one (on the left) has a constant liquid supply, the second one (on the right) has a leak. The liquid level of each tank is denoted by variable h_1 and h_2 .

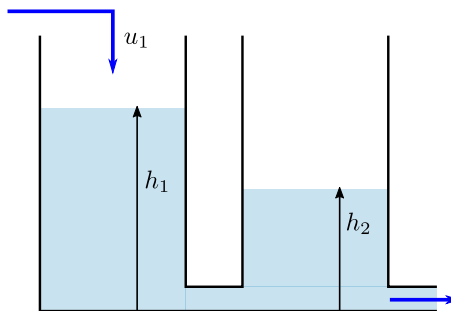


FIGURE 1

A dynamical model, based on Bernoulli and Torricelli principles, is given by :

$$\begin{cases} \dot{h}_1 = \frac{1}{\rho S_1} u_1 - \frac{S_{12}}{S_1} \text{sign}(h_1 - h_2) \sqrt{2g|h_1 - h_2|} \\ \dot{h}_2 = \frac{S_{12}}{S_2} \text{sign}(h_1 - h_2) \sqrt{2g|h_1 - h_2|} - \frac{S_{20}}{S_2} \sqrt{2g h_2} \end{cases}$$

with $u_1 = 10 \text{ kg/s}$ a constant inflow rate, $S_1 = S_2 = 0.1 \text{ m}^2$ and $S_{12} = S_{20} = 0.005 \text{ m}^2$ are cross-sectional areas, $\rho = 1000 \text{ kg/m}^3$ is water density, $g = 10 \text{ m/s}^2$ is the gravitational acceleration. The model becomes :

$$\begin{cases} \dot{h}_1 = 0.1 - 0.05 \text{sign}(h_1 - h_2) \sqrt{20|h_1 - h_2|} \\ \dot{h}_2 = 0.05 \text{sign}(h_1 - h_2) \sqrt{20|h_1 - h_2|} - 0.05 \sqrt{20 h_2} \end{cases}$$

1. What is (are) the equilibrium point(s) of the system ?
2. Calculate the linearized model around the (each) equilibrium point.
3. What is (are) his (their) nature? In each case, how liquid levels h_1 and h_2 will evolve ?
4. Sketch the phase portrait of the system around the equilibrium point(s). Particularly, use eigenvectors.
5. Simulate the system with MATLAB/Simulink and plot some trajectories for different initial conditions.

Recall of the sign function

$$\text{sign}(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \end{cases}$$