

## TD 5

### *Lyapunov and estimation of the basin of attraction* (stability analysis)

---

**Objective :** Apply the Lyapunov method to a nonlinear system and illustrate the concept of invariant set. The Lyapunov function is then used to estimate the basin of attraction of the equilibrium point.

Consider the system described by :

$$\begin{cases} \dot{x}_1 = x_1^3(x_1x_2 - 1) + x_1(x_1x_2 - 1 + x_2^2) \\ \dot{x}_2 = -x_2 \end{cases}$$

1. Show that the origin is the unique equilibrium point.
2. Study its stability properties using the first Lyapunov method.
3. Show that the set  $\Omega = \{x \in \mathbb{R}^2 : x_1x_2 \geq 2\}$  is an invariant set.
4. From the previous question, what can we say about the stability of the origin ?
5. Consider the scalar function  $V(x) = x_1^2 + x_2^2$ . What is the condition on  $x_1$  and  $x_2$  ensuring that  $V$  is a Lyapunov function ?
6. Determine the disk  $D_r$  of maximal radius  $r$  included in the basin of attraction of the origin.