TD8 : Feedback linearization

(State feedback stabilization)

Objective : Design a state feedback control that linearizes a second order nonlinear system and enables to place the poles of the closed-loop system.

Consider the following second order system

$$\begin{cases} \dot{x}_1 &= x_1 + \dfrac{x_2}{1+x_1^2} \\ \dot{x}_2 &= -x_2 + u \end{cases}$$

where u is is the control input.

1. What is (are) the equilibrium point(s) of the system when there is no control u ?

2. Still for $u = 0$, is the system stable ?

3. Applying the change of variable $z = T(x) = \begin{pmatrix} x_1 \\ x_1 + \frac{x_2}{1+x_1^2} \end{pmatrix}$, show that the initial system can be re-expressed as
$$\dot{z} = Az + B\Big(\gamma(x)u - \psi(x)\Big)$$
where A and B are constant matrices, γ and ψ are scalar functions of the state x.

4. What state feedback control law enables to linearized the system and then allows to place the poles for the z-coordinates closed-loop system ? Give the values for gains k_1 and k_2 so as to have the two poles equal to -1.

5. Simulate the system with the above control law on Simulink.