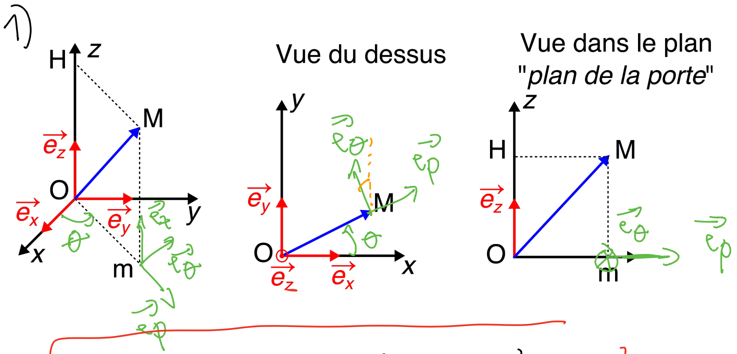
Exercice 1

1)



2) $\vec{e}_\rho \left| \begin{array}{l} \cos\theta \\ \sin\theta \\ 0 \end{array} \right. \qquad \vec{e}_\theta \left| \begin{array}{l} -\sin\theta \\ \cos\theta \\ 0 \end{array} \right. \qquad \vec{e}_z \left| \begin{array}{l} 0 \\ 0 \\ 1 \end{array} \right.$

3) $$\frac{d\vec{e}_\rho}{dt} = \left| \begin{array}{l} \frac{d}{dt}(\cos\theta) \\ \frac{d}{dt}(\sin\theta) \\ 0 \end{array} \right. = \left| \begin{array}{l} -\frac{d\theta}{dt}\sin\theta \\ \frac{d\theta}{dt}\cos\theta \\ 0 \end{array} \right. = \frac{d\theta}{dt}\vec{e}_\theta = \underline{\underline{\dot{\theta}\vec{e}_\theta}}$$

$$\frac{d\vec{e}_\theta}{dt} = -\dot{\theta}\vec{e}_\rho \qquad \frac{d\vec{e}_z}{dt} = \vec{0}$$

4) $$\vec{OM} = \rho\vec{e}_\rho + z\vec{e}_z$$

$$\boxed{\vec{OM} = \left| \begin{array}{l} \rho \\ 0 \\ z \end{array} \right.}$$

$$\frac{d\vec{OM}}{dt} = \dot{\rho}\vec{e}_\rho + \rho\frac{d\vec{e}_\rho}{dt} + \dot{z}\vec{e}_z + z\frac{d\vec{e}_z}{dt}$$

$$\vec{v}_{M/R} = \dot{\rho}\vec{e}_\rho + \rho\dot{\theta}\vec{e}_\theta + \dot{z}\vec{e}_z$$

$$\boxed{\vec{v}_{M/R} = \left| \begin{array}{l} \dot{\rho} \\ \rho\dot{\theta} \\ \dot{z} \end{array} \right.}$$

$$d\vec{OM} = \vec{dl} = d\rho\,\vec{e}_\rho + \rho\,d\theta\,\vec{e}_\theta + dz\,\vec{e}_z$$

$$\boxed{\vec{dl} \left| \begin{array}{l} d\rho \\ \rho\,d\theta \\ dz \end{array} \right.}$$

Exercice 2

1) $z = OH = r\cos\theta$

$OP = r\sin\theta$

$$M \left| \begin{array}{l} x \\ y \\ z \end{array} \right. = \left| \begin{array}{l} OP\cos\varphi \\ OP\sin\varphi \\ z \end{array} \right. = \left| \begin{array}{l} r\sin\theta\cos\varphi \\ r\sin\theta\sin\varphi \\ z \end{array} \right.$$

$\vec{u} = \cos\varphi \, \vec{u}_x + \sin\varphi \, \vec{u}_y$

$\vec{u}_\varphi \perp \vec{u}$ et il fait un angle de $(\varphi + \pi/2)$ avec l'axe (Ox)

* $\vec{u}_\varphi = \cos(\varphi + \pi/2) \, \vec{u}_x + \sin(\varphi + \pi/2) \, \vec{u}_y$

$= -\sin(\varphi) \, \vec{u}_x + \cos(\varphi) \, \vec{u}_y$

$$\boxed{\vec{u}_\varphi \begin{vmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{vmatrix}}$$

* $\vec{u}_r = \sin\theta \, \vec{u} + \cos\theta \, \vec{u}_z$

$= \sin\theta \left[\cos\varphi \, \vec{u}_x + \sin\varphi \, \vec{u}_y\right] + \cos\theta \, \vec{u}_z$

$$\boxed{\vec{u}_r \begin{vmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{vmatrix}}$$

$\vec{u}_\theta \perp \vec{u}_r$

* $\vec{u}_\theta = \cos\theta \, \vec{u} - \sin\theta \, \vec{u}_z$

$= \cos\theta \left[\cos\varphi \, \vec{u}_x + \sin\varphi \, \vec{u}_y\right] - \sin\theta \, \vec{u}_z$

$$\boxed{\vec{u}_\theta \begin{vmatrix} \cos\theta\cos\varphi \\ \cos\theta\sin\varphi \\ -\sin\theta \end{vmatrix}}$$

2) $\vec{u}_r = \begin{vmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{vmatrix}$

$$\boxed{\frac{\partial \vec{u}_r}{\partial \theta} = \begin{vmatrix} \cos\theta\cos\varphi \\ \cos\theta\sin\varphi \\ -\sin\theta \end{vmatrix} = \vec{u}_\theta}$$

$$\frac{\partial \vec{u}_r}{\partial \varphi} = \begin{vmatrix} -\sin\theta\sin\varphi \\ \sin\theta\cos\varphi \\ 0 \end{vmatrix} = \sin\theta \begin{vmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{vmatrix}$$

$$\boxed{\frac{\partial \vec{u}_r}{\partial \varphi} = \sin\theta \, \vec{u}_\varphi}$$

$$\boxed{\frac{\partial \vec{u}_\theta}{\partial \theta} = \begin{vmatrix} -\sin\theta\cos\varphi \\ -\sin\theta\sin\varphi \\ -\cos\theta \end{vmatrix} = -\vec{u}_r}$$

$$\boxed{\frac{\partial \vec{u}_\theta}{\partial \varphi} = \begin{vmatrix} -\cos\theta\sin\varphi \\ \cos\theta\cos\varphi \\ 0 \end{vmatrix} = \cos\theta \, \vec{u}_\varphi}$$

$$\frac{\partial \vec{u}_\varphi}{\partial \theta} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} \quad \frac{\partial \vec{u}_\varphi}{\partial \varphi} = \begin{vmatrix} -\cos\varphi \\ -\sin\varphi \\ 0 \end{vmatrix} = \begin{vmatrix} -\cos\varphi[\sin^2\theta + \cos^2\theta] \\ -\sin\varphi[\sin^2\theta + \cos^2\theta] \\ 0 \end{vmatrix}$$

$$= -\sin\theta\,\vec{u}_r - \cos\theta\,\vec{u}_\theta$$

$$\boxed{\frac{\partial \vec{u}_\varphi}{\partial \theta} = -\sin\theta\,\vec{u}_r - \cos\theta\,\vec{u}_\theta}$$

3) $$d\vec{u}_r = \cancel{\frac{\partial \vec{u}_r}{\partial r}\,dr} + \frac{\partial \vec{u}_r}{\partial \theta}\,d\theta + \frac{\partial \vec{u}_\varphi}{\partial \varphi}\,d\varphi$$

$$\boxed{d\vec{u}_r = d\theta\,\vec{u}_\theta + d\varphi\,\sin\theta\,\vec{u}_\varphi}$$

$$d\vec{u}_\theta = \cancel{\frac{\partial \vec{u}_\theta}{\partial r}\,dr} + \frac{\partial \vec{u}_\theta}{\partial \theta}\,d\theta + \frac{\partial \vec{u}_\theta}{\partial \varphi}\,d\varphi$$

$$\boxed{d\vec{u}_\theta = -\,d\theta\,\vec{u}_r + d\varphi\,\cos\theta\,\vec{u}_\varphi}$$

$$d\vec{u}_\varphi = \cancel{\frac{\partial \vec{u}_\varphi}{\partial r}\,dr} + \cancel{\frac{\partial \vec{u}_\varphi}{\partial \theta}\,d\theta} + \frac{\partial \vec{u}_\varphi}{\partial \varphi}\,d\varphi$$

$$\boxed{d\vec{u}_\varphi = -\sin\theta\,d\varphi\,\vec{u}_r - \cos\theta\,d\varphi\,\vec{u}_\theta}$$

4) $\frac{d\vec{u}_r}{dt} = \vec{\omega} \wedge \vec{u}_r$? voir le cours

$\vec{\omega}$: vecteur rotation (additivité)

1ère rotation φ autour de $\vec{Oz}$

$$\vec{\omega}_1 = \frac{d\varphi}{dt} \vec{u}_z \qquad \vec{u}_z = [\cos\theta \, \vec{u}_r - \sin\theta \, \vec{u}_\theta]$$

2nde rotation θ autour de $\vec{u}_\varphi$

$$\vec{\omega}_2 = \frac{d\theta}{dt} \vec{u}_\varphi$$

$$\boxed{\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2 = \frac{d\varphi}{dt}[\cos\theta \, \vec{u}_r - \sin\theta \, \vec{u}_\theta] + \frac{d\theta}{dt} \vec{u}_\varphi}$$

$$\boxed{\vec{\omega} = \left| \begin{array}{l} \cos\theta \frac{d\varphi}{dt} \\ -\sin\theta \frac{d\varphi}{dt} \\ \frac{d\theta}{dt} \end{array} \right.}$$

$$\frac{d\vec{u}_r}{dt} = \vec{\omega} \wedge \vec{u}_r = \left| \begin{array}{l} \cos\theta \, \dot{\varphi} \\ -\sin\theta \, \dot{\varphi} \\ \dot{\theta} \end{array} \right. \wedge \left| \begin{array}{l} 1 \\ 0 \\ 0 \end{array} \right. = \left| \begin{array}{l} 0 \\ \dot{\theta} \\ +\sin\theta \, \dot{\varphi} \end{array} \right.$$

$$* \quad \frac{d\vec{u}_\theta}{dt} = \vec{\omega} \wedge \vec{u}_\theta = \left| \begin{matrix} \cos\theta\,\dot{\varphi} \\ -\sin\theta\,\dot{\varphi} \\ \dot{\theta} \end{matrix} \right. \wedge \left| \begin{matrix} 0 \\ 1 \\ 0 \end{matrix} \right. = \left| \begin{matrix} -\dot{\theta} \\ 0 \\ \cos\theta\,\dot{\varphi} \end{matrix} \right.$$

$$* \quad \frac{d\vec{u}_\varphi}{dt} = \vec{\omega} \wedge \vec{u}_\varphi = \left| \begin{matrix} \cos\theta\,\dot{\varphi} \\ -\sin\theta\,\dot{\varphi} \\ \dot{\theta} \end{matrix} \right. \wedge \left| \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \right. = \left| \begin{matrix} -\sin\theta\,\dot{\varphi} \\ -\cos\theta\,\dot{\varphi} \\ 0 \end{matrix} \right.$$

5) $\vec{V} \left| \begin{matrix} V_r \\ V_\theta \\ V_\varphi \end{matrix} \right.$

$$\frac{d\vec{V}}{dt} = \dot{V}_r \vec{u}_r + V_r \frac{d\vec{u}_r}{dt} + \dot{V}_\theta \vec{u}_\theta + V_\theta \frac{d\vec{u}_\theta}{dt} + \dot{V}_\varphi \vec{u}_\varphi + V_\varphi \frac{d\vec{u}_\varphi}{dt}$$

$$\frac{d\vec{V}}{dt} = \left| \begin{matrix} \dot{V}_r - \dot{\theta} V_\theta - \sin\theta\,\dot{\varphi}\,V_\varphi \\ \dot{V}_\theta + \dot{\theta} V_r - \cos\varphi\,\dot{\varphi}\,V_\varphi \\ \dot{V}_\varphi + \sin\theta\,\dot{\varphi}\,V_r + \cos\theta\,\dot{\varphi}\,V_\theta \end{matrix} \right.$$