INSA, 5gmm

Lifetime Data Analysis

Assimilation Exercises 2: Useful Lifetime Distributions

Exercice 1 *(Exponential distribution)*

Suppose that X has an exponential distribution with parameter λ. Using only the definition of an exponential distribution (the expression of its p.d.f. or its Reliability function), prove the following equalities:

1. $\lambda(x) = \lambda, \forall x \geq 0$;
2. $R(x|x_0) = R(x), \forall (x, x_0) \in \mathbb{R}^+ \times \mathbb{R}^+$;
3. $\mathcal{L}(\tau_{x_0}) = \mathcal{L}(X), \forall x_0 \in \mathbb{R}^+$.
4. $m(x) = \frac{1}{\lambda}, \forall x \geq 0$.

Consider again questions 1 and 4 for the shifted exponential distribution $\mathcal{E}_{x_0}(\lambda)$.

Exercice 2 *(Conditional Reliability function)*

Select the software you prefer (R, Python, Excel...) and plot the figures of Slide 49.

Exercice 3 *(Weibull distribution)*

1. Prove the result given in the second remark of Slide 53: the minimum of n i.i.d. r.v. with same Weibull $W(\alpha, \beta)$ distribution has a Weibull $W(\alpha/n^{1/\beta}, \beta)$ distribution.
2. What could you say about the shape of the hazard function of a Weibull distribution, in function of the shape parameter β. Prove it!