Lifetime Data Analysis

Assimilation Exercises 5: Non Parametric Inference with complete observations

Exercice 1 *(Empirical Estimation)*

With the software of your choice, plot the estimates of the Reliability function $R(\cdot)$, the hazard rate function $\lambda(\cdot)$ and the cumulative hazard rate function $\Lambda(\cdot)$, based on the observations given in Slide 96 (competing risks dataset).

Exercice 2 *(Equivalence of the two expressions of the Reliability estimator)*

The aim here is to show the equivalence given in Slide 101.

1. Write the expression of the estimator of the hazard rate function at the observed value of the ist order statistics $x_{(i)}$.
2. Show that the expression
$$\widehat{R}(j) = \prod_{i=0}^{j} \left(1 - \widehat{\lambda}(i)\right)$$
leads to
$$\widehat{R}(x_{(i)}) = \prod_{j \leq i} \left(1 - \widehat{\lambda}(x_{(j)})\right), \text{ for } i = 1, \ldots, n.$$
3. Using the expression of $\widehat{\lambda}(x_{(i)})$ given in question 1 and developing this product, show that we have:
$$\widehat{R}(x_{(i)}) = \frac{n-i}{n}.$$
4. Show that this is nothing but the empirical estimator given in Slide 94.

Exercice 3 *(Asymptotic Results)*

If the proofs of the asymptotic results given in Slide 103 are outside the scope of this lectures, you should be able to prove simpler related results. We consider the expression of the estimator of the Reliability function given in Slide 94.

1. Prove[1] that we have, *almost surely* (*a.s.*):
$$\widehat{R}_n(x) \to R(x), \text{ when } n \to +\infty.$$
2. Prove[2] that, for all x in $\mathbb{R}^+$:
$$\sqrt{n}\left(\widehat{R}_n(x) - R(x)\right) \xrightarrow{\mathcal{L}} N\left(0, F(x)R(x)\right),$$
when n tends to $+\infty$.
3. Deduce from this later result, the asymptotic $(1-\alpha)$ confidence interval for $R(x)$ given in Slide 104.

[1] Use one of the most well known results in Probability and Statistics...
[2] Use the other one...