4AE-SE / 4AE FISA
Analyse des systèmes nonlinéaires

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TD 2

Position control of a DC motor with an input saturation (phase plane)

Objective : Study the orbits of a closed-loop linear system with a saturation function at the control input.

Consider the closed-loop system described by :

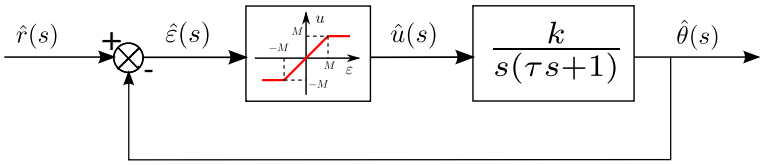

FIGURE 1

where the saturation function $\text{sat}(\cdot)$ is defined by

$$u = \text{sat}(\varepsilon) = \begin{cases} M & \text{if } \varepsilon > M \\ \varepsilon & \text{if } |\varepsilon| < M \\ -M & \text{if } \varepsilon < -M \end{cases}$$

Such a transfert function, a first order system and an integrator, is a typical model to describe the dynamic of the angular position θ of DC motors. The saturation phenomenon is due to the amplifier limitation to generate the input voltage u. Some parameters are set : $M = 0.5$, $\tau = 1$ and $r = 4$ *rad* (constant reference).

1. Choosing $x_1 = r - \theta$ and $x_2 = -\dot{\theta}$ as state variables, give the state space model of the above closed-loop system.

2. What are the equilibrium points ?

3. In the case of $|\varepsilon| < M$, what is the qualitative behavior around the origin w.r.t. k ?

4. In the general case, sketch the phase portrait when $k = 0.1$ and $k = 3$.

5. Check your calculations with MATLAB/Simulink. Try different values of gain k.