4AE-SE / 4AE FISA
Analyse des systèmes nonlinéaires

TD 3

Lyapunov method (stability analysis)

Objective : Apply Lyapunov method and LaSalle principle to assess the stability of a nonlinear system.

Exercise 1

Consider the following second order system :

$$\begin{cases} \dot{x}_1 = -x_1 - x_2 \\ \dot{x}_2 = 2x_1 - x_2^3 \end{cases}$$

1. Show that the origin is the unique equilibrium point.
2. Let consider the Lyapunov function candidate $V(x) = ax_1^2 + bx_2^2$, where a and b are free positive scalars. After having verified that V is a positive definite function, compute its time-derivative to analyze the stability of the system.
3. Is the stability result local or global ?

Exercise 2

Consider the following differential equation :

$$\ddot{y} + (1 - y^2)\dot{y} + y = 0$$

1. Write the state-space model of the differential equation.
2. What is (are) the equilibrium point(s) ?
3. Prove the asymptotic stability of the equilibrium point with the Lyapunov function candidate $V(x) = x_1^2 + x_2^2$. Draw on the phase plane the set $\mathcal{D}$ such that $\dot{V}(x) \leq 0$.
4. Is the stability result local or global ?
5. How do level curves $V(x) = c$ look like in the phase plane ? Draw the larger level curve that is included in $\mathcal{D}$. Then define the set $\mathcal{D}_S$ as an estimation of the region of attraction.