INSA TOULOUSE

TD 4

Lyapunov for linear systems (stability analysis)

Objective : Apply Lyapunov method to assess the stability of linear systems.

Exercise 1

Consider the following linear system :

$$\dot{x} = \begin{bmatrix} -1 & -1 \\ 4 & 0 \end{bmatrix} x$$

1. Let consider the general quadratic Lyapunov function candidate $V(x) = x^T P x$, where $P \in \mathbb{R}^{2\times 2}$ is a symmetric matrix. What are the conditions for V to be a positive definite function ?

2. Give the condition to be solved to analyze the stability of linear systems of the form $\dot{x} = Ax$.

3. Solve the Lyapunov equation, with $Q = \mathbb{1}$, to analyze the stability of the above system.

Exercise 2

Consider the following linear system :

$$\dot{x} = \begin{bmatrix} -1 & 2 \\ 6 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

1. Firstly, let consider the system without input, that is $u = 0$. Analyze the stability of the system with the Lyapunov method.

2. Now, the system is controlled with an output feedback : $u = -ky + y_c$, where k is a gain to be designed. y_c is the reference input. What condition on k ensures the closed-loop system stability ?

3. Verify your condition with MATLAB/Simulink.