4AE-SE / 4AE FISA
Analyse des systèmes nonlinéaires

INSA TOULOUSE

TD 7

Van der Pol oscillator

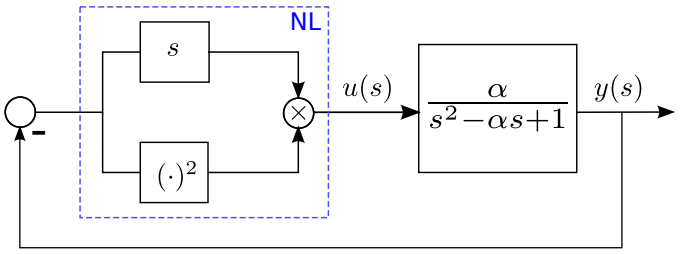
(describing function)

Objective : Reformulate a nonlinear system as a linear feedback system with a separated nonlinear block and then apply the describing function method to predict the presence of a limit cycle.

Consider the differential equation of the form of a Van der Pol oscillator :

$$\ddot{y} + \alpha(y^2 - 1)\dot{y} + y = 0$$

1. Show that the above equation can be represented as a feedback system of the form below, containing a linear block and a nonlinear one.



2. Assuming there are oscillations (limit cycle) and $\varepsilon = -y = A\sin(\omega t)$. Express the signal u et derive the complex equivalent gain $N(A, \omega)$ approximating the nonlinearity.

3. Give the conditions for the existence of a limit cycle.

4. Setting ω to the value obtained in the previous question, sketch the graph $-1/N(A)$ in the complex plane. Check your previous result with MATLAB. Is the limit cycle stable ?

5. Determine a state space representation of $G(s)$ and simulate the feedback system for some initial conditions ($\neq 0$). Take $\alpha = 1$. Observe both the output time response and the phase plane trajectory.