Elements de Corrigé

Embrayage centrifuge

A. A1. le plan Oxz est plan de symetrie ($O \in$ axe de rotation)

le solide étant homogene $G_1 \in$ plan (Oxz) $\Rightarrow$ $y_{G_1} = 0$ (1)

A2. la matrice est donnee en G_1

G_1xz est plan de symetrie $\Rightarrow$ $D_1 = F_1 = 0$

La matrice est donc de la forme $\begin{pmatrix} A_1 & 0 & -E_1 \\ 0 & B_1 & 0 \\ -E_1 & 0 & C_1 \end{pmatrix}$ (1)

A3. Huyghens entre O et G_1

$$[I_{O,(1),B_1}] = [I_{G_1,(1),B_1}] + [I_{O,\, m_1 \to G_1}]$$

$$\vec{OG_1} = \begin{pmatrix} -0{,}052 \\ 0 \\ 5{,}12 \end{pmatrix}$$

$m_1 = 0{,}149$ kg

$$[I_{O,m_1 \to G_1}] = \begin{pmatrix} m_1 Z_G^2 & 0 & -m_1 X_G Z_G \\ 0 & m_1(X_G^2 + Z_G^2) & 0 \\ -m_1 X_G Z_G & 0 & m_1(X_G^2) \end{pmatrix}$$

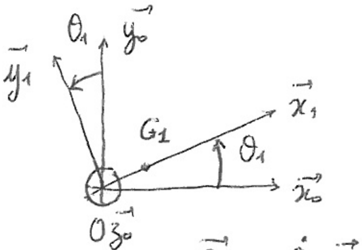
$X_G \ll Z_G$

$$= \begin{pmatrix} 3{,}9\ 10^{-6} & 0 & -0{,}04\ 10^{-6} \\ 0 & 3{,}6\ 10^{-6} & 0 \\ -0{,}04\ 10^{-6} & 0 & \varepsilon \end{pmatrix}$$ (2)

on retrouve bien l'égalité de Huyghens.

B. B1.

Nature du mouvement = rotation / $O\vec{z}_0$



$$\vec{\Omega}_{1/0} = \dot{\theta}_1 \vec{z}_0$$

$$\left\{\mathcal{C}_i(1/R_0)\right\} = \left\{ \begin{matrix} \vec{R_c}(1/R_0) \\ \vec{\sigma}_{G_1}(1/R_0) \end{matrix} \right\}_{G_1}$$

$$\vec{R_c}(1/R_0) = m_1 \vec{V}_{G_1,1/R_0} = -m_1 X_G \vec{y}_1 \cdot \dot{\theta}_1$$

$$\vec{\sigma}_{G_1}(1/R_0) = \left[I_{G_1}(1) \right] \left\{ \vec{\Omega}(1/R_0) \right\} = \begin{pmatrix} -E_1 \dot{\theta}_1 \\ 0 \\ C_1 \dot{\theta}_1 \end{pmatrix}_{B_1}$$

(1)

B2.

$$\begin{cases} \vec{R_d}(1/R_0) = \dfrac{d \vec{R_c}(1/R_0)}{dt} = -m_1 X_G \ddot{\theta}_1 \vec{y}_1 + m_1 X_G \dot{\theta}_1^2 \vec{x}_1 \\ \vec{\delta}_{G_1}(1/R_0) = \dfrac{d \vec{\sigma}_{G_1}(1/R_0)}{dt}\Big|_{R_0} = \begin{pmatrix} -E_1 \ddot{\theta}_1 \\ -E_1 \dot{\theta}_1^2 \\ C_1 \ddot{\theta}_1 \end{pmatrix}_{B_1} \end{cases}$$

(1)

B3.

$$\vec{\delta}_{O}(1/R_0) = \vec{\delta}_{G_1}(1/R_0) + \underbrace{\left(\vec{OG_1} \wedge \vec{R_d}(1/R_0) \right)}$$

$$\begin{pmatrix} -X_G \\ 0 \\ Z_G \end{pmatrix} \wedge \begin{pmatrix} m_1 X_G \dot{\theta}_1^2 \\ -m_1 X_G \ddot{\theta}_1 \\ 0 \end{pmatrix} = \begin{pmatrix} m_1 X_G Z_G \ddot{\theta}_1 \\ m_1 X_G Z_G \dot{\theta}_1^2 \\ m_1 X_G^2 \ddot{\theta}_1 \end{pmatrix}_{B_1}$$

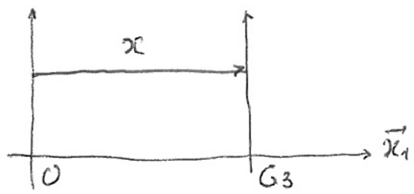
(1)

remarque : on retrouve l'expression de B2 avec les termes de la matrice de la figure 3b (en O)

$$\vec{\delta}_O(1/R_0) = \begin{pmatrix} (-E_1 + m_1 X_G Z_G) \ddot{\theta}_1 \\ (-E_1 + m_1 X_G Z_G) \dot{\theta}_1^2 \\ (C_1 + m_1 X_G^2) \ddot{\theta}_1 \end{pmatrix}_{B_1}$$

B4.

$$\vec{\Omega}_{3/0} = \vec{\Omega}_{3/1} + \vec{\Omega}_{1/0} = \dot{\theta}_1 \vec{z}_0$$



$$\vec{V}_{G_3,3/0} = \underbrace{\vec{V}_{G_3,3/1}}_{\dot{x}\,\vec{x}_1} + \underbrace{\vec{V}_{G_3,1/0}}_{x \cdot \dot{\theta}_1 \vec{y}_1}$$

$$\vec{V}_{G_3,3/0} = \dot{x}\,\vec{x}_1 + x \dot{\theta}_1 \vec{y}_1$$

(1)

On deduit $\vec{a}_{G3,3/0} = \dfrac{d\vec{V}_{G3,3/0}}{dt} = \ddot{x}\,\vec{x}_1 + x\ddot{\theta}_1\,\vec{y}_1 - x\dot{\theta}_1^2\,\vec{x}_1 + 2\dot{x}\dot{\theta}_1\,\vec{y}_1$ 1

B5. Torseur cinetique de (3) en O

$$\vec{R_c}(3/R_0) = m_3\,\vec{V}_{G3,3/0} = m_3\,(\dot{x}\,\vec{x}_1 + x\dot{\theta}_1\,\vec{y}_1)$$

$$\vec{\sigma}_{G_3}(3/R_0) = [I_{G_3,(3)}]\,.\{\Omega(3/0)\} = \begin{vmatrix} 0 \\ 0 \\ C_3\dot{\theta}_1 \end{vmatrix}$$

$$\vec{\sigma}_O(3/R_0) = \vec{\sigma}_{G_3}(3/R_0) + \{\underbrace{\vec{OG_3}}_{x.\vec{x}_1} \wedge \vec{R_c}(3/R_0)\} = \begin{vmatrix} 0 \\ 0 \\ C_3\dot{\theta}_1 + m_3x^2\dot{\theta}_1 \end{vmatrix}_{B_1}$$ 1

B6. Torseur dynamique

$$\vec{R_d}(3/R_0) = m_3\,\vec{a}_{G3,3/0} = m_3\{\ddot{x}\,\vec{x}_1 + (x\ddot{\theta}_1\,\vec{y}_1 - x\dot{\theta}_1^2\,\vec{x}_1) + 2\dot{x}\dot{\theta}_1\,\vec{y}_1\}$$

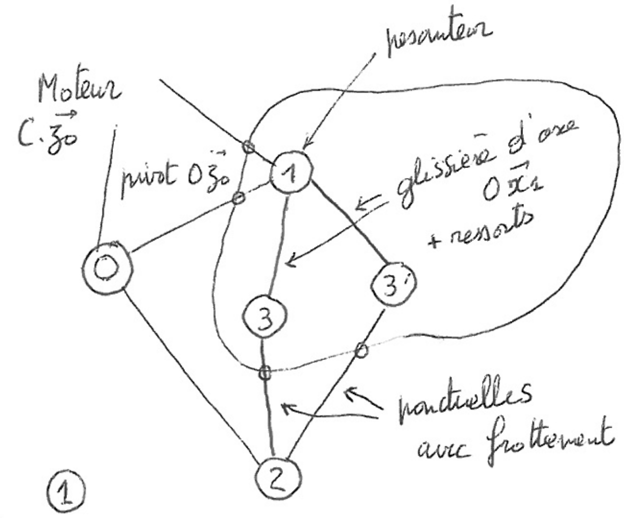
$$\vec{\delta}_O(3/R_0) = \frac{d\vec{\sigma}_O(3/R_0)}{dt} = \begin{pmatrix} 0 \\ 0 \\ (C_3 + m_3x^2)\ddot{\theta}_1 + 2m_3x\dot{x}\dot{\theta}_1 \end{pmatrix}$$ 1

B7. $\vec{\delta}_O(\Sigma/R_0) = \vec{\delta}_O(1/R_0) + 2\,\vec{\delta}_O(3/R_0)$

on obtient $\vec{\delta}_O(\Sigma/R_0) = \begin{pmatrix} (-E_1 + m_1X_GZ_G)\ddot{\theta}_1 \\ (-E_1 + m_1X_GZ_G)\dot{\theta}_1^2 \\ (C_1 + 2C_3 + m_1X_G^2 + 2m_3x^2)\ddot{\theta}_1 + 4m_3x\dot{x}\dot{\theta}_1 \end{pmatrix}$

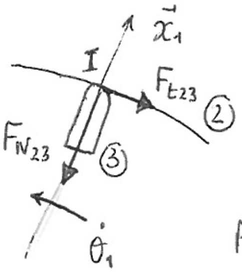
1

C. Contact avec glissement



②

- on isole le (Σ)
- Pb spatial
- BAM ext
  - couple moteur $C(t)\,\vec{z_0}$
  - liaison pivot $0 \to 1$ $\quad \left\{ \begin{array}{c|c} X_{01} & L_{01} \\ Y_{01} & M_{01} \\ Z_{01} & 0 \end{array} \right\}_{O B_0}$ ①
  - pontuelles
  - pesanteur négligée



glissement $\Rightarrow$ $F_{t23} = f\, F_{N23}$

F_{t23} s'oppose à la vitesse de glissement de 3/2 $\quad \vec{V}_{glis\, I, 3/2} = R\dot{\theta}_1\, \vec{y_1}$

①

- $\sum \vec{M}_{O\, ext \to \Sigma} = \vec{\delta}_{O(\Sigma/R_0)}$

$$\begin{cases} L_{01} = (-E_1 + m_1 X_G Z_G)\,\ddot{\theta}_1 \\ M_{01} = (-E_1 + m_1 X_G Z_G)\,\dot{\theta}_1^{\,2} \\ C(t) - R(F_{t23} + F'_{t23}) = \left\{ (C_1 + 2C_3 + m_1 X_G^2 + 2m_3 x^2)\,\ddot{\theta}_1 + 4 m_1 x \dot{x} \dot{\theta}_1 \right\} \end{cases}$$

$$= \left(C_1 + 2C_3 + m_1 X_G^2 + 2 m_3 R^2\right)\ddot{\theta}_1$$

②

rigueur scientifique ②