Partie 1

Q1a $M_t = M_v + M_c$ AN : $M_t = 57\,000\ kg$ (0,5)

Q1b

$$a < \vec{FG}\cdot\vec{x_1} < b$$

$$M_t\,\vec{FG} = M_v\,\vec{FG_v} + M_c\,\vec{FG_c}$$

$$\vec{FG}\,\vec{x_1} = \frac{M_v}{M_t}c + \frac{M_c}{M_t}d$$

$$\left(a - \frac{M_v}{M_t}c\right)\frac{M_t}{M_c} < d < \left(b - \frac{M_v}{M_t}c\right)\frac{M_t}{M_c} \qquad (1)$$

AN : $-0{,}12 < d < 1{,}78\ m$ (0,5)

Partie 2

Q2a. 2 plans de symétrie $\perp$ passant par $Axyz$ ⇒ $D = E = F = 0$

solide de révolution $/A\vec{z_1}$ ⇒ $A = B$ (1)

$J_{2x} = J_{2y}$

Q2.b

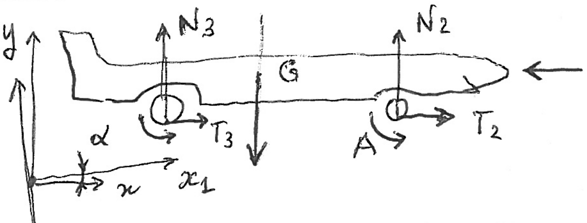
$$[I_{A,b_1}(2)] = \begin{pmatrix} 2(J_{2x} + m_2e^2) & 0 & 0 \\ 0 & 2(J_{2y} + m_2e^2) & 0 \\ 0 & 0 & 2J_{2z} \end{pmatrix}$$

$A(2) = B(2)$ (1,5)

Partie 3

$\dot{x} = v \qquad \dot\theta_2 = \dot\theta_3 = \text{cste}$

Q3a



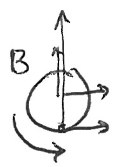
BAM ext
- pes → 1 : $-Mg\vec{y}$
- vent → 1 : $-k_1(\dot{x} - v_0)^2\,\vec{x_1}$
- sol → 2
- sol → 3 : $\begin{Bmatrix} T_i & 0 \\ N_i & 0 \\ 0 & k_iN_i \end{Bmatrix}_{I_i}$

$\sum \vec{F}_{ext \to \Sigma} = \vec{0}$

$$\begin{cases} T_3 + T_2 - k_1(\dot{x} - v_0)^2 - Mg\sin\alpha = 0 \\ N_3 + N_2 - Mg\cos\alpha = 0 \end{cases} \qquad (0,5)$$

$$\cancel{k_1}N_3 + k_2N_2 - (\ell + L)N_3 + Mg(\cos\alpha . L + h\sin\alpha) + k_1(\dot{x} - v_0)^2 . h = 0 \qquad (0,5)$$

on isole (3)



$\sum \vec{M}_{B\,ext \to 3} = \vec{0} \qquad R_3 . T_3 + k_3N_3 = 0 \qquad (0,5)$

Q3b existence du contact $N_2 > 0 \quad N_3 > 0$

non glissement $|T_2| < f N_2 \quad |T_3| < f N_3$ (0,5)

Q3c $N_2 = Mg\cos\alpha - N_3 = Mg\cos\alpha - \left\{ \dfrac{Mg(L\cos\alpha + h\sin\alpha)}{(L+\ell)} + k_1(\dot{x}-v_0)^2 \cdot \dfrac{h}{(L+\ell)} \right\}$

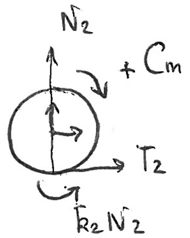
(1)

$T_2 = Mg\sin\alpha + k_1(\dot{x}-v_0)^2 + \dfrac{k_3}{R_3}\left\{ \dfrac{Mg(L\cos\alpha + h\sin\alpha)}{(L+\ell)} + k_1(\dot{x}-v_0)^2 \dfrac{h}{(L+\ell)} \right\}$

(1)

$f \geq \dfrac{T_2}{N_2}$

Q3d on isole (2)



(1)

$-C_m + T_2 . R + k_2 N_2 = 0$

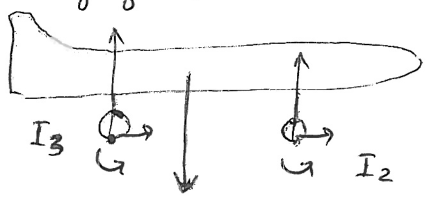
$C_m = R . T_2 + k_2 N_2$

Q3e le calcul de T_2 et N_2 donne $\dfrac{T_2}{N_2} = 0{,}62$ $(< 0{,}7)$ (1)

Partie 4 $(\alpha = 0 \quad k_1 = 0)$

Q4.a RSG $\dot{x} + R_2\dot{\theta}_2 = 0 \quad \dot{x} + R_3\dot{\theta}_3 = 0$ (1)

Q4 b $\sum \vec{F}_{ext} = \overrightarrow{Rd(\Sigma/R_g)} = M . \ddot{x}\, \vec{x}_1$ (M_2 et M_3 negligées)



(0,5) $\begin{cases} T_2 + T_3 = M\ddot{x} \\ N_2 + N_3 - Mg = 0 \end{cases}$

$\sum \overrightarrow{M}_{I_2\, ext \to \Sigma} = \vec{\delta}_{I_2}(\Sigma/R_g)$

$k_2 N_2 + MgL - N_3(L+\ell)$

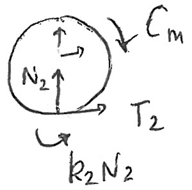
$\vec{\delta}_{I_2, 1/R_g} = -M\ddot{x} . h$

$\vec{\delta}_{I_2, 2/R_g} = J_2\ddot{\theta}_2 + 0$

$\vec{\delta}_{I_2, 3/R_g} = J_3\ddot{\theta}_3 + 0$

$k_3 N_3 + k_2 N_2 + MgL - N_3(L+\ell) = J_2\ddot{\theta}_2 + J_3\ddot{\theta}_3 - Mh\ddot{x}$ (1)

on isole (2)



$$T_2 R_2 + k_2 N_2 - C_m = J_2 \ddot{\theta}_2$$

1

on isole (3) par analogie

$$T_3 R_3 + k_3 N_3 = J_3 \ddot{\theta}_3$$

Q4c

$$|T_2| \leq f N_2$$

0,5

Q4d

$$N_3(L+\ell) = MgL + \underbrace{\left[Mh + \left(\frac{J_2}{R_2} + \frac{J_3}{R_3}\right)\right]}_{A} \ddot{x} - k_2 N_2 \Rightarrow N_3$$

$$N_3 = \frac{MgL + A\ddot{x}}{(L+\ell)} - \cancel{\frac{k_2 N_2}{(L+\ell)}}$$

$$N_2 = Mg - \frac{MgL}{(L+\ell)} - \frac{A\ddot{x}}{(L+\ell)} = \frac{Mg\ell}{(L+\ell)} - \frac{A\ddot{x}}{(L+\ell)}$$

1

$$N_2 > 0 \qquad \ddot{x} < \frac{Mg\ell}{A}$$

1

$$T_3 = -\frac{k_3}{R_3} N_3 - \frac{J_3}{R_3^2} \ddot{x}$$

$$T_2 = M\ddot{x} - T_3 = M\ddot{x} + \frac{k_3}{R_3} N_3 + \frac{J_3}{R_3^2} \ddot{x}$$

$$T_2 = \left[M + \frac{J_3}{R_3^2} + \frac{k_3}{R_3}\frac{A}{(L+\ell)}\right] \ddot{x} + \frac{k_3}{R_3}\frac{MgL}{(L+\ell)}$$

1

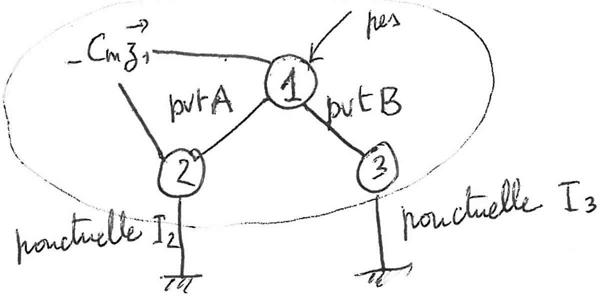
$$\frac{T_2}{N_2} \leq f$$

1

on trouve $\frac{T_2}{N_2} = 0,65$

1,5

Partie 5



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on isole l'Σ

$$P_{ext \to \Sigma, R_g} + P_{int} = \left(\frac{d\,Ec(\Sigma/R_g)}{dt}\right)_{/R_g}$$

- RsG en I_2 et I_3

il reste $k_2 N_2 . \dot{\theta}_2 + k_3 N_3 \dot{\theta}_3$

~~- pesanteur~~ $\cancel{\theta}$

$$-C_m \dot{\theta}_2 = \frac{C_m}{R_2}\dot{x}$$ 0,5

$$Ec(\Sigma) = Ec(1) + Ec(2) + Ec(3)$$
$$= \frac{1}{2}M\dot{x}^2 + \frac{1}{2}J_2\dot{\theta}_2^2 + \frac{1}{2}J_3\dot{\theta}_3^2$$ 0,5

$$C_m\frac{\dot{x}}{R_2} = M\dot{x}\ddot{x} + \frac{J_2}{R_2^2}\dot{x}\ddot{x} + \frac{J_3}{R_3^2}\dot{x}\ddot{x} + Mg\sin\alpha\,\dot{x} + k_1(\dot{x}-v_0)^2\dot{x}$$

en négligeant k_2 et k_3

- pente $-Mg\sin\alpha . \dot{x}$
- air $-k_1(\dot{x}-v_0)^2\dot{x}$

$$C_m = R_2\left\{\left(M + \frac{J_2}{R_2^2} + \frac{J_3}{R_3^2}\right)\ddot{x} + Mg\sin\alpha + k_1(\dot{x}-v_0)^2\right\}$$

en negligeant k_2 et k_3

1