MEMS accelerometer

The aim of this exercice is to modelize a dynamic system, a MEMS accelerometer, with Newtonien and Lagrangian approachs.

1. Airbag

An airbag is a vehicle safety device. It is an occupant restraint system consisting of a flexible fabric envelope or cushion designed to inflate rapidly during an automobile collision. Most designs are inflated through pyrotechnic means and can only be operated once. Each airbag device is typically activated with one or more pyrotechnic devices, commonly called an initiator or electric match. The electric match, which consists of an electrical conductor wrapped in a combustible material, activates with a current pulse between 1 to 3 amperes in less than 2 milliseconds. When the conductor becomes hot enough, it ignites the combustible material, which initiates the gas generator.

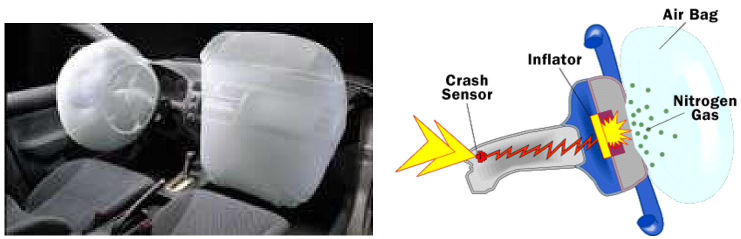


Figure n° 1 – Airbag principle

Figure n° 2 – Crash test

Frontal offset crash testing (Fig 2) has gained acceptance worldwide as an assessment of the frontal crashworthiness of vehicles. The crush characteristics of vehicle front structures have typically been modeled as simple mass-spring systems. We want to calculate some numerical values for a car characterized by a M=1500 kg mass and a vehicle front structure stiffness of K=900 kN/m.

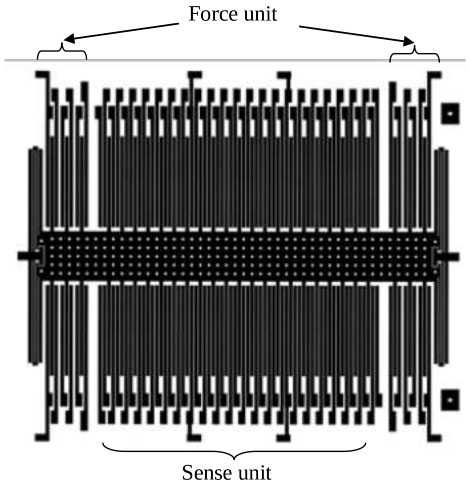
1. Thanks to energy conservation theorem give the maximum deformation x_{max}, the maximum force F_{max} and the maximal acceleration a_{max} for a 64 km/h crash test.
2. Find again these results by applying second Newton laws. Determine natural frequency of the system.

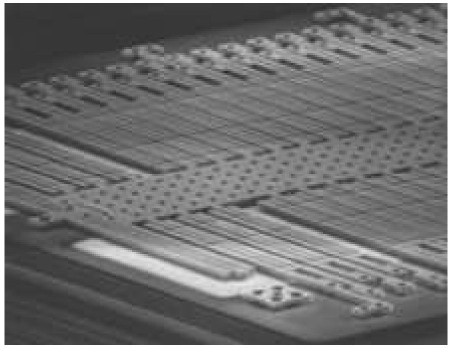
2. MEMS accelerometer: Newtonian approach

The measurement of acceleration for the crash detection is realized thanks MEMS (Micro Electro Mechanical System) accelerometer. The Fig 3 is a example of a micro-machined stand-alone accelerometer, the Analog Devices ADXL150, which consists of a mass spring system as well as a system to measure displacement and the appropriate signal conditioning circuitry. The surface micromachined sensor element is made by depositing polysilicon on a sacrificial oxide layer that is then etched away leaving the suspended sensor element. Fig. 4 is a simplified view of the sensor structure. Self-test (testing part of Fig 3) is activated by an electrostatic force acts on the beam equivalent to approximately 20% of full-scale acceleration input, and thus a proportional voltage change appears on the output pin. When activated, the self-test feature exercises both the entire mechanical structure and the electrical circuitry. This testing part can also be used for closed-loop force feedback control of the accelerometer.

The basic physical principle behind this accelerometer (as well as many others), is that of a mass spring system (Fig. 5) with :

- m, the seismic mass
- k, the stiffness of the spring
- f, viscous friction
- x_b, position of the sensor base
- x_m, position of the seismic mass
- C_1 and C_2, two capacities used to mesure $x=x_m-x_b$





Partial view of one end of the sensor

Figure n° 3 – Silhouette plots of ADXL150 sensor. Axis of motion is horizontal

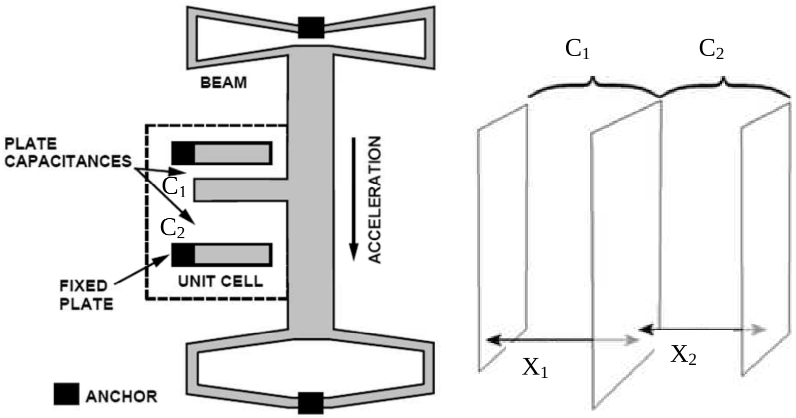
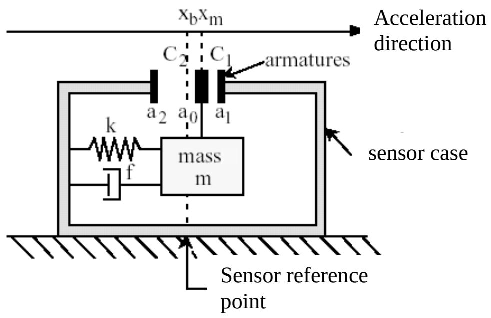


Figure n° 4 – The dual Capacitor system used to measure displacement in the Analog Devices ADXL50 accelerometer



m = 2,2.10^{-10} kg
k = 5,4 N/m
f = 6.9.10^{-6} N.s/m

Figure 5 - Mass-Spring system used for measuring acceleration

1. Locate the *K*, *f*, *m*, C_1 and C_2 elements of Fig. 5 diagram on the Fig. 4 diagram.
2. Apply Newton's second law of motion on mass *m*. Write the differential equation with x_b and *x* ($x = x_m - x_b$) where x_b is position of the sensor base and x_m, position of the seismic mass.
3. Give the transfer function between $a_b=d^2x_b/dt^2$ (input) and *x* (output) with this form :

$$F(p) = \frac{X(p)}{A_b(p)} = \frac{F_0}{1 + 2\xi\frac{p}{\omega_0} + \frac{p^2}{\omega_0^2}}$$

   Give expressions and value of static gain F_O, pulsation ω_0, damping coefficient ξ.
4. Link, in steady state, the sensor case acceleration $a_b=d^2x_b/dt^2$ with the measure *x*. Give the proportionnal ratio. Compare the characteristical frequency of the accelerometer whit the mass-spring system of the crash test.

3. MEMS accelerometer: Lagrangian approach

1. Find previous equation with Lagrangian approach.

The Lagragian approach can also be used to calculate parameter values, as stiffness suspension. Potential energy of the Lagrangian is function of elastic deformation of the structure. For a traction/compression beam, Fig. 6, the link between stress and deformation is:

$$\sigma = E.\varepsilon$$

Where:

- $\sigma = E.\varepsilon$ is the stress;
- $\varepsilon = \frac{\Delta L}{L}$ is the deformation;
- E is the Young modulus equal to 150 GPa for Silicon.

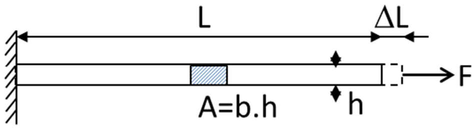


Figure 6 – Traction/compression beam

2. Demonstrate that volumic elastic energy is $\frac{1}{2}\sigma\varepsilon$.
3. Demonstrate that for flexural deformation as illustrated Fig. 7, the elastic energy can be calculated with :

$$E_p = \iiint \frac{1}{2} E y^2 \left(\frac{\partial^2 u(x)}{\partial x^2}\right)^2 dv$$

where *y* and *u(x)* are defined in Fig. 7 and *v* is the volume. Assumption: *x* and *x+dx* sections are orthogonal to medium line of the beam.

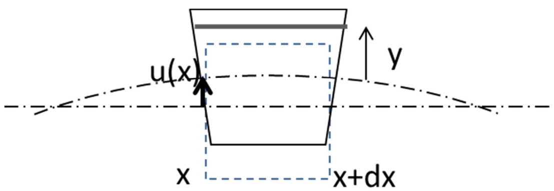


Figure 7 – Flexural deformation

4. Propose a third order polynomial approximation of the deformation of one arm (Length *L*, height *h*, thickness *b*) of a U-spring suspension (see Fig 8.) in function of the proof mass displacement U_0.
5. Show how to include the elastic deformation energy into the potential part of the Lagrangian function.
6. Derivate the Lagrange equation of the system and give expressions (do not solve the integral) of the suspension stiffness *K*.

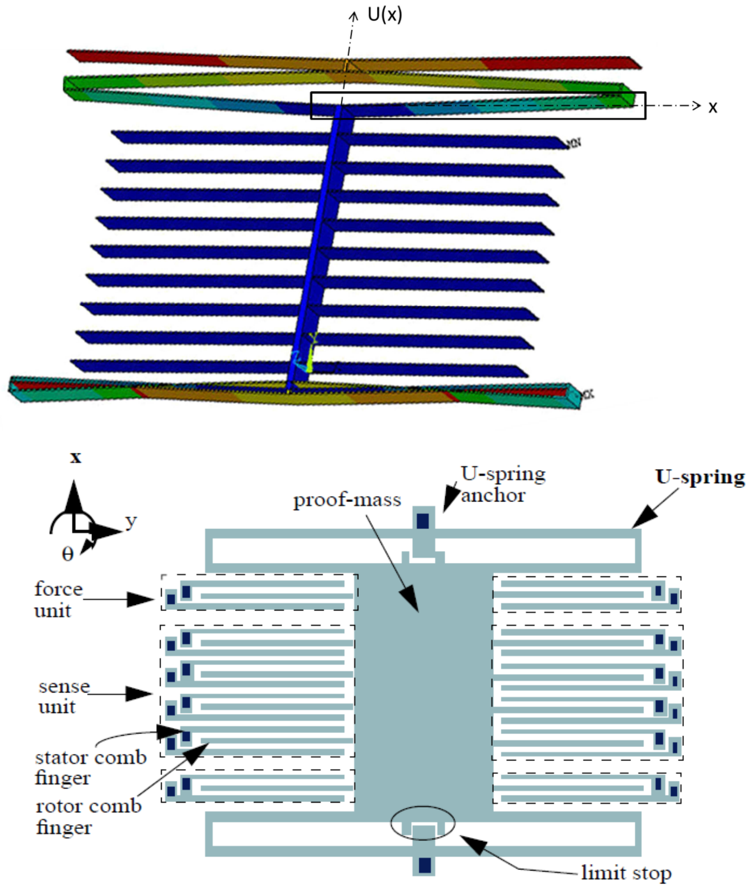


Figure 8 – U-spring deformation