IMACS 3A / MECANIQUE

MEMS Accelerometer

(I) Airbag

1° $E_c = \frac{1}{2} M v^2 = \frac{1}{2} 1500 \cdot (17,8)^2 = 237 \text{ kJ}$ car $64 \text{ km/h} = 17,8 \text{ m/s}$

↳ $E_p = \frac{1}{2} k \Delta x^2 \Rightarrow \Delta x_{max} = \sqrt{\frac{2 E_c}{k}} = 72 \text{ cm}$

mais aussi

$E_p = \frac{1}{2} \frac{F^2}{k} \Rightarrow F_{max} = \sqrt{2 E_c k} = 6,53 \cdot 10^5 \text{ N}$

↳ et $F_{max} = M \frac{dv}{dt} \Rightarrow a_{max} = \frac{F_{max}}{M} = 435 \text{ m.s}^{-2}$ soit $44 g$

2° $M \frac{dv}{dt} = M \frac{d^2 x}{dt^2} = -kx$ soit $M \frac{d^2 x}{dt^2} + kx = 0$

Si on suppose $x(t)$ de la forme $x(t) = X_0 \cdot e^{rt}$

$x''(t) = r^2 X_0 e^{rt}$

$M r^2 + k = 0 \quad r = \pm i \sqrt{\frac{k}{M}}$

On a donc $x(t) = x_0 \sin(\omega t)$ où $\omega = \sqrt{\frac{k}{M}} = 24,5 \text{ rad.s}^{-1} = 3,9 \text{ Hz}$

$\dot{x}(t) = \underbrace{\omega x_0} \cos(\omega t)$

↳ v soit $x_0 = \frac{v}{\omega_0} = 0,72 \text{ m}$ on retrouve bien Δx_{max}

$\ddot{x}(t) = -\underbrace{\omega^2 x_0} \sin(\omega t)$

↳ $a_{max} = 435 \text{ m.s}^{-2}$ on retrouve bien a_{max}

II) MEMS Accelerometer : Newtonian approach

2° $m \frac{d^2 x_m}{dt} = -k(x_m - x_b) - f(\dot{x}_m - \dot{x}_b)$

$m \frac{d^2 (\overbrace{x_m - x_b}^{x})}{dt^2} + k(\underbrace{x_m - x_b}_{x}) + f(\underbrace{\dot{x}_m - \dot{x}_b}_{\dot{x}}) = -m \underbrace{\frac{d^2 x_b}{dt^2}}_{a_b}$

3° $\frac{X}{A_b} = \frac{-m}{k + fp + mp^2} = \frac{-m/k}{1 + f/k \, p + m/k \, p^2} \qquad \omega_0 = \sqrt{\frac{k}{m}}$

$= \frac{F_0}{1 + 2\xi \frac{p}{\omega_0} + \frac{p^2}{\omega_0^2}}$ où $\frac{2\xi}{\omega_0} = \frac{f}{k}$ soit $\xi = \frac{\omega_0}{2} \frac{f}{k}$

$F_{max} = \frac{m}{k}$

4. $\frac{x}{a_b} = |F_{max}| = \frac{m}{k} = 4,07 \cdot 10^{-11} \text{ m/m.s}^{-2}$ soit $0,17 \mu m$ pour exo précédent

$\omega_0 = \sqrt{\frac{k}{m}} = 25 \text{ kHz} \gg 3,9 \text{ Hz} \rightarrow$ on est en régime établi.

(II) Lagrangian approach

1° On a $\mathcal{L} = E_c - E_p = \frac{1}{2} m \dot{x}_m^2 - \frac{1}{2} k (x_m - x_b)^2$

/. x_m : $\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_m} - \frac{\partial \mathcal{L}}{\partial x_m} = m \ddot{x}_m + k (x_m - x_b) = \frac{\partial W}{\partial x_m}$ où $W = -\beta(\dot{x}_m - \dot{x}_b)^2$

2° $E_p = \frac{1}{2} k \Delta l^2$ où $k = \frac{AE}{L}$ car $\sigma = \frac{F}{A} = E \frac{\Delta l}{L}$

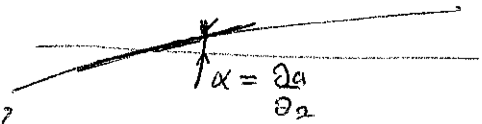
d'où $E_p = \frac{1}{2} \frac{AE}{L} \Delta l^2 = \frac{1}{2} \underbrace{AL}_{\text{volume}} \cdot \overbrace{E \frac{\Delta L}{L}}^{\sigma} \underbrace{\frac{\Delta l}{L}}_{\varepsilon} = \frac{1}{2} \underbrace{AL}_{\text{volume}} \cdot \sigma \cdot \varepsilon$ cqfd

3° l'allongement d'une fibre de la poutre située à la distance y de la ligne médiane (ligne neutre) :

ΔL = déplacement horiz en $x+dx$ − déplacement horiz en x

$\frac{\partial u}{\partial x}(x+dx)\, y$ $\qquad$ $\frac{\partial u}{\partial x}(x)\, y$

soit $\frac{\Delta L}{L} = \frac{\Delta L}{dx} = \frac{\partial^2 u}{\partial x^2} y$ $\qquad$ ↳ car

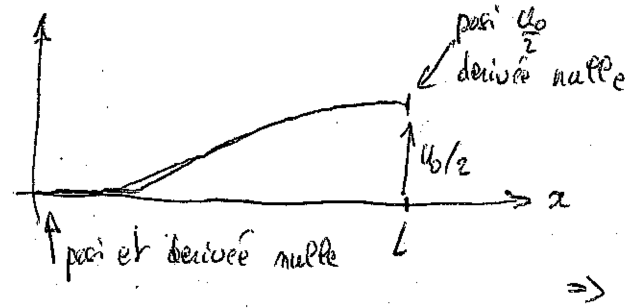


d'où

$E_p = \iiint \frac{dE}{d\sigma} d\sigma$ où $\frac{dE}{d\sigma} = \frac{1}{2} E \varepsilon^2 = \frac{1}{2} E \left(\frac{\partial^2 u}{\partial x^2}\right)^2 y^2$

$$\boxed{E_p = \iiint \frac{1}{2} E \left(\frac{\partial^2 u}{\partial x^2}\right)^2 y^2 \, d\sigma}$$

4° $u(x) = ax^3 + bx^2 + cx + d$ est de la forme



avec $u(0) = 0 \to \boxed{d=0}$

$u'(0) = 0 \to \boxed{c=0}$

$u'(L) = 0 \to 3aL^2 + 2bL = 0$

$u(L) = \frac{u_0}{2} \to aL^3 + bL^2 = \frac{u_0}{2}$

$\Rightarrow b = -\frac{3aL}{2} \Rightarrow aL^3 - \frac{3a}{2}L^3 = \frac{u_0}{2}$

$\boxed{b = \frac{3}{2}\frac{u_0}{L^2}}$ $\qquad$ $\boxed{a = -\frac{u_0}{L^3}}$

soit $u(x) = ax^3 + bx^2 = -\frac{u_0}{L^3}x^3 + \frac{3}{2}\frac{u_0}{L^2}x^2$

$u(x) = u_0 \left(\frac{x}{L}\right)^2 \left(\frac{3}{2} - \left(\frac{x}{L}\right)\right)$

$u'(x) = 3ax^2 + 2bx$

$u''(x) = 6ax + 2b = -\frac{6u_0}{L^3}x + \frac{3u_0}{L^2} = \frac{3u_0}{L^2}\left(-2\left(\frac{x}{L}\right) + 1\right)$

5.° $\mathcal{L} = E_c - E_p = \frac{1}{2} m \dot{u}_0^2 - \frac{1}{2} k u_0^2$

6.°

$E_c \rightarrow \frac{1}{2} m \dot{u}_0(t)$

$E_p \rightarrow \iiint \frac{\partial E_p}{\partial v} dv$ sur les 8 bras deformés

$$= 8 . \iiint_{1 \text{ bras}} \frac{\partial E}{\partial v} dv = 8 . \frac{1}{2} \int_{x=0}^{x=L} \int_{y=-\frac{h}{2}}^{y=\frac{h}{2}} E \left(\frac{\partial^2 u}{\partial x^2}\right)^2 y^2 \, b \, dx \, dy$$

$$= 4 . b \underbrace{\int_{y=-\frac{h}{2}}^{y=\frac{h}{2}} y^2 \, dy}_{\frac{bh^3}{12}} . E . u_0^2(t) . \frac{9}{L^4} \frac{1}{6} \underbrace{\left[\ldots \right]_0^1 L}_{\text{par chgt variable } u = \frac{x}{L} \text{ soit } \frac{L}{3}}$$

$$= 4 \frac{bh^3}{12} . E \, u_0^2(t) . \frac{3}{L^3} = \frac{1}{2} k \, u_0^2$$

D'où $\boxed{k = 2 \frac{bh^3 . E}{L^3}}$