

1) Test terrain:

- Raideur de la butée flexible qui attache la tuyère au bâti
- Poids de la tuyère (quand elle est braquée)
- Force générée par l'actionneur

Pendant le vol:

- Effort généré par l'excentrement du vecteur poussé
- Efforts aéros

2) 
$$y \ddot{\theta} = l(\theta) F_a - Mg L \sin \theta - K \theta$$



3) a)

$$I_{yy} = \int (x^2 + z^2) dm$$

$$dm = \frac{M}{\text{surface cône}} \cdot dA = \frac{M \sin \alpha}{\pi R^2} dA$$

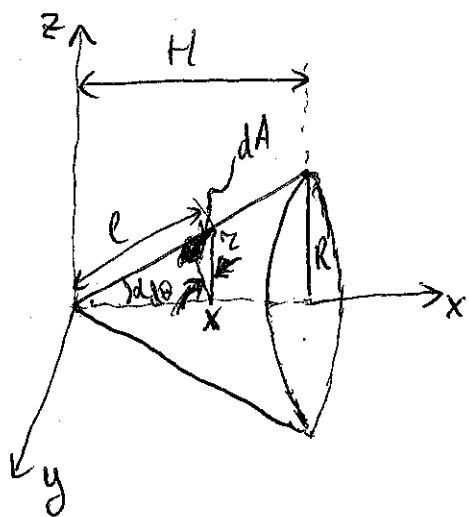
$$dA = r d\theta dl$$

$$\frac{z}{x} = \frac{R}{H} \Rightarrow z = \frac{R}{H} x$$

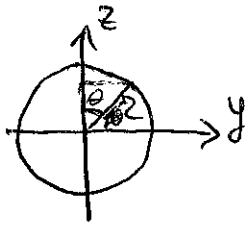
$$l = \frac{r}{\sin \alpha} = \frac{R}{H \sin \alpha} x \Rightarrow dl = \frac{R}{H \sin \alpha} dx$$

$$dA = \frac{R}{H} x d\theta \frac{R}{H \sin \alpha} dx = x \frac{R^2}{H^2 \sin \alpha} d\theta dx$$

$$dm = \frac{M \sin \alpha}{\pi R^2} \cdot x \frac{R^2}{H^2 \sin \alpha} d\theta dx = x \frac{M}{\pi H^2} dx d\theta$$



$$I_{yy} = \int_0^{2\pi} \int_0^H (x^2 + z^2) x \frac{M}{\pi H^2} dx d\theta$$



$$z = R \cos \theta = x \frac{R}{H} \cos \theta$$

$$I_{yy} = \frac{M}{\pi H^2} \iint (x^2 + x^2 \frac{R^2}{H^2} \cos^2 \theta) x dx d\theta$$

$$= \frac{M}{\pi H^2} \left( \int_0^{2\pi} \int_0^H x^3 dx d\theta + \frac{R^2}{H^2} \int_0^{2\pi} \int_0^H x^3 \cos^2 \theta dx d\theta \right)$$

$$= \frac{M}{\pi H^2} \left( 2\pi \frac{H^4}{4} + \frac{R^2}{H^2} \cdot \frac{H^4}{4} \int_0^{2\pi} \frac{1}{2} (1 + \cos(2\theta)) d\theta \right)$$

Rappel:  $\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$

$$= \frac{M}{\pi H^2} \left( \frac{2\pi H^4}{4} + \frac{R^2 H^2}{4} \cdot \frac{1}{2} \left( 2\pi + \underbrace{\frac{\sin(2\theta)}{2}}_{=0} \Big|_0^{2\pi} \right) \right)$$

$$I_{yy} = \frac{M}{4} (2H^2 + R^2)$$

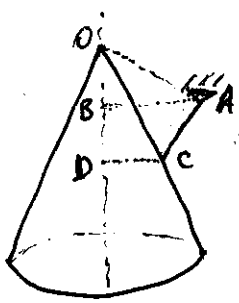
A.N.  $I_{yy} = \frac{1616}{4} (2 \cdot 2,5^2 + 1^2) = 5454 \text{ kg} \cdot \text{m}^2$

b)  $G = (x_G, y_G, z_G) - ?$

$x_G = z_G = 0$  évident

$$x_G = \frac{\int x dm}{M} = \frac{1}{M} \int_0^{2\pi} \int_0^H x \cdot x \frac{M}{\pi H^2} dx d\theta = \frac{1}{\pi H^2} \int_0^{2\pi} \int_0^H x^2 dx d\theta = \frac{1}{\pi H^2} \frac{H^3}{3} \cdot 2\pi = \frac{2}{3} H$$

c)



Travaux virtuels:

$$F \cdot dx = C d\theta \quad \text{avec } x = AC$$

$$F \frac{dx}{d\theta} = C$$

$l$  - bras de levier

$$x = AC = \sqrt{OC^2 + OA^2 - 2 \cdot OC \cdot OA \cdot \cos(\angle AOC)}$$

$$OC = \sqrt{OD^2 + DC^2} = \sqrt{(0,197 + 0,947)^2 + 0,967^2} = 1,498 \text{ m}$$

$$OA = \sqrt{OB^2 + BA^2} = \sqrt{1,368^2 + 0,197^2} = 1,382 \text{ m}$$

$$\angle AOC = \angle AOB - \angle COD + \theta$$

$$\tan \angle AOB = \frac{AB}{OB} \Rightarrow \angle AOB = \arctan \frac{1,368}{0,197} = 81,8^\circ$$

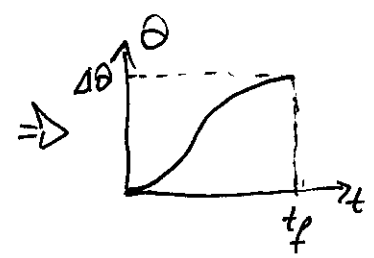
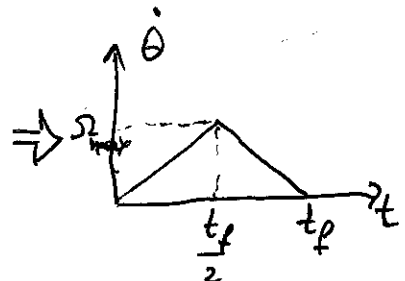
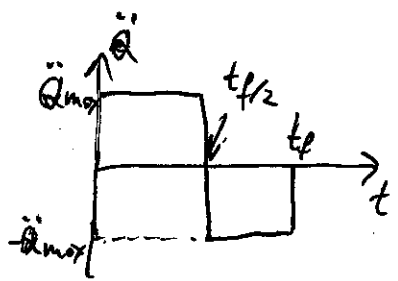
$$\tan \angle COD = \frac{DC}{DO} \Rightarrow \angle COD = \arctan \frac{0,967}{0,197 + 0,947} = 40,2^\circ$$

$$\angle AOC = 81,8^\circ - 40,2^\circ + \theta = 41,6^\circ + \theta$$

$$dx = \frac{1}{2} \frac{2 \cdot OC \cdot OA \cdot \sin(41,6^\circ + \theta)}{\sqrt{OC^2 + OA^2 - 2 \cdot OC \cdot OA \cdot \cos(41,6^\circ + \theta)}} d\theta$$

$$l = \frac{dx}{d\theta} = \frac{2,07 \sin(41,6^\circ + \theta)}{\sqrt{4,154 - 4,14 \cos(41,6^\circ + \theta)}}$$

4) Pour un déplacement  $\Delta\theta$  dans un temps minimal, l'accélération doit avoir la forme:



$$\theta(t) = \underbrace{\theta_0}_{=0} + \underbrace{\dot{\theta}_0}_{=0} t + \frac{\ddot{\theta}(t) t^2}{2}$$

si  $\Delta\theta = \theta(t_f)$  alors  $\frac{\Delta\theta}{2} = \theta\left(\frac{t_f}{2}\right) = \frac{\ddot{\theta}_{\max} \left(\frac{t_f}{2}\right)^2}{2}$

$$\ddot{\theta}_{\max} = \frac{4 \cdot \Delta\theta}{t_f^2}$$

comme  $\theta \in (-5,7^\circ \div +5,7^\circ)$  on approxime  $\sin \theta \approx \theta$

entre  $\theta = 5,5^\circ$  et  $\theta = 0^\circ$   $l$  varie seulement de 2,4 cm  $\Rightarrow l = \text{constant}$   
 $l = 1,33 \text{ m}$

i)  $F.l = J \ddot{\theta}_{\max} + (Mg l + k) \theta$  pour  $\theta \in (0 \div \frac{\Delta\theta}{2})$

ii)  $F.l = -J \ddot{\theta}_{\max} + (Mg l + k) \theta$  pour  $\theta \in (\frac{\Delta\theta}{2} \div \Delta\theta)$   
*Alors on trouve*

i)  $F.l = \max$  pour  $\theta = \frac{\Delta\theta}{2}$  } effort max pour  $\theta = \frac{\Delta\theta}{2}$

ii)  $F.l = \min$  pour  $\theta = \frac{\Delta\theta}{2}$   
 $\uparrow$  car on freine

$$F_{\max} = \frac{J \ddot{\theta}_{\max} + (Mg l + k) \theta}{l} = \frac{J \frac{4 \cdot \Delta\theta}{t_f^2} + (Mg l + k) \frac{\Delta\theta}{2}}{l}$$

a) Scenario 1:  $F_{\max} \approx 5,1 \text{ kN}$

b) Scenario 2:  $F_{\max} \approx 4,1 \text{ kN}$