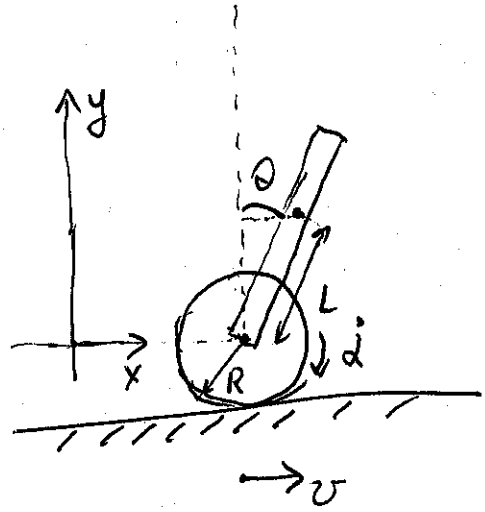
1) Coordonnés généralisées

α et θ

roue tige



2) $E_p = M_h g L(1+\cos\theta)$

$E_c = E_c^{roues} + E_c^{tige}$

$E_c^{roue} = 2\times\left(\frac{1}{2} J_w \dot{\alpha}^2 + \frac{1}{2} M_w v^2\right) = 2\left(\frac{1}{2} J_w \dot{\alpha}^2 + \frac{1}{2} M_w R^2 \dot{\alpha}^2\right)$

car 2 roues

$E_c^{tige} = \frac{1}{2} J_h \dot{\theta}^2 + \frac{1}{2} M_h v_{tige}^2$ $\qquad v_{tige}^2 = v_x^2 + v_y^2$

$v_y = \frac{d}{dt}(L\cos\theta) = -L\dot{\theta}\sin\theta$

$v_x = \frac{d}{dt}(X + L\sin\theta) = \dot{x} + L\dot{\theta}\cos\theta = R\dot{\alpha} + L\dot{\theta}\cos\theta$

$v_{tige}^2 = L^2\dot{\theta}^2\sin^2\theta + R^2\dot{\alpha}^2 + 2R\dot{\alpha}L\dot{\theta}\cos\theta + L^2\dot{\theta}^2\cos^2\theta$

$= L^2\dot{\theta}^2 + R^2\dot{\alpha}^2 + 2R\dot{\alpha}L\dot{\theta}\cos\theta$

$E_c = \frac{1}{2}\cdot 2J_w\dot{\alpha}^2 + \frac{1}{2}2M_w R^2\dot{\alpha}^2 + \frac{1}{2}J_h\dot{\theta}^2 + \frac{1}{2}M_h\left(L^2\dot{\theta}^2 + R^2\dot{\alpha}^2 + 2R\dot{\alpha}L\dot{\theta}\cos\theta\right)$

$E_c = \frac{1}{2}\left(2J_w + 2M_w R^2 + M_h R^2\right)\dot{\alpha}^2 + \frac{1}{2}\left(J_h + M_h L^2\right)\dot{\theta}^2 + M_h R L\dot{\alpha}\dot{\theta}\cos\theta$

3) $$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot q}\right) - \frac{\partial \mathcal{L}}{\partial q} = \frac{\delta W}{\delta q}$$ avec $q = \{\alpha, \theta\}$ et $\mathcal{L} = E_c - E_p$

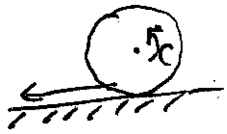
$$\frac{\partial \mathcal{L}}{\partial \dot\theta} = (J_h + M_h L^2)\dot\theta + M_h R L \dot\alpha \cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot\theta}\right) = (J_h + M_h L^2)\ddot\theta + M_h R L \ddot\alpha \cos\theta - M_h R L \dot\alpha \sin\theta\,\dot\theta$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = - M_h R L \dot\alpha \dot\theta \sin\theta + M_h g L \sin\theta$$

$$(J_h + M_h L^2)\ddot\theta + M_h R L \ddot\alpha \cos\theta - \cancel{M_h R L \dot\alpha \dot\theta \sin\theta} + \cancel{M_h R L \dot\alpha \dot\theta \sin\theta} - M_h g L \sin\theta = 0$$

$$\boxed{(J_h + M_h L^2)\ddot\theta + M_h R L \ddot\alpha \cos\theta - M_h g L \sin\theta = 0}$$



$F_2 = C_2 (M_h + M_w) g$

$\frac{\delta W}{\delta \alpha} = C_f = F_2 \cdot R = R C_2 (M_h + M_w) g$

$$\frac{\partial \mathcal{L}}{\partial \dot\alpha} = (2J_w + 2M_w R^2 + M_h R^2)\dot\alpha + M_h R L \dot\theta \cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot\alpha}\right) = (2J_w + 2M_w R^2 + M_h R^2)\ddot\alpha + M_h R L \ddot\theta \cos\theta - M_h R L \dot\theta^2 \sin\theta$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = 0$$

$$\boxed{(2J_w + 2M_w + M_h R^2)\ddot\alpha + M_h R L \ddot\theta \cos\theta - M_h R L \dot\theta^2 \sin\theta = C - R C_2 g (M_h + M_w)}$$

4) Equation de mouvement de la forme :

$$a\ddot{\theta} + b\ddot{\alpha}\cos\theta - c\sin\theta = 0$$

$$d\ddot{\alpha} + b\ddot{\theta}\cos\theta - b\dot{\theta}^2\sin\theta = \text{Couple} - e$$