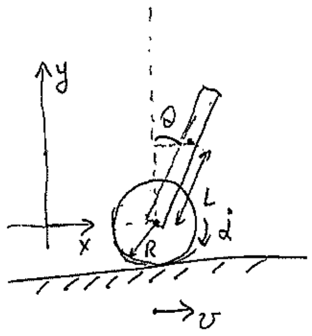
IMACS 3 / MECHANIQUE

TD 6 : Segway

1) Coordonnés généralisées

α et θ

roue   tige



(roue) $E_p = 0\,J$ car $G_{roue} \equiv$ centre du repère

(barre tige) 2) $E_p = M_\ell g L(\cos\theta)$

$E_c = E_c^{roues} + E_c^{tige}$ $v = R\dot\alpha$ (Roulement ss glissement)

- $E_c^{roue} = 2\times\left(\frac{1}{2}J_w\dot\alpha^2 + \frac{1}{2}M_w v^2\right) = 2\left(\frac{1}{2}J_w\dot\alpha^2 + \frac{1}{2}M_w R^2\dot\alpha^2\right)$

car 2 roues

- $E_c^{tige} = \frac{1}{2}J_\ell\dot\theta^2 + \frac{1}{2}M_\ell v_{tige}^2$ $v_{tige}^2 = v_x^2 + v_y^2$

$OT|_y = L\cos\theta$

$OT|_x = x + L\sin\theta$

$v_y = \frac{d}{dt}(L\cos\theta) = -L\dot\theta\sin\theta$

$v_x = \frac{d}{dt}(x + L\sin\theta) = \dot x + L\dot\theta\cos\theta = R\dot\alpha + L\dot\theta\cos\theta$

$v_{tige}^2 = L^2\dot\theta^2\sin^2\theta + R^2\dot\alpha^2 + 2R\dot\alpha L\dot\theta\cos\theta + L^2\dot\theta^2\cos^2\theta$ $= 1$

$= L^2\dot\theta^2 + R^2\dot\alpha^2 + 2R\dot\alpha L\dot\theta\cos\theta$

$E_c = \frac{1}{2}\cdot 2J_w\dot\alpha^2 + \frac{1}{2}2M_w R^2\dot\alpha^2 + \frac{1}{2}J_\ell\dot\theta^2 + \frac{1}{2}M_\ell\left(L^2\dot\theta^2 + R^2\dot\alpha^2 + 2R\dot\alpha L\dot\theta\cos\theta\right)$

roue   tige

$$E_c = \frac{1}{2}\left(2J_w + 2M_w R^2 + M_\ell R^2\right)\dot\alpha^2 + \frac{1}{2}\left(J_\ell + M_\ell L^2\right)\dot\theta^2 + \cancel{\tfrac{2}{2}}\cdot M_\ell R L\dot\alpha\dot\theta\cos\theta$$

3) $\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot q}\right) - \frac{\partial\mathcal{L}}{\partial q} = \frac{\delta W}{\delta q}$ avec $q = \{\alpha, \theta\}$ et $\mathcal{L} = E_c - E_p$

- $\frac{\partial\mathcal{L}}{\partial\dot\theta} = (J_\ell + M_\ell L^2)\dot\theta + M_\ell R L\dot\alpha\cos\theta$

$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot\theta}\right) = (J_\ell + M_\ell L^2)\ddot\theta + M_\ell R L\ddot\alpha\cos\theta - M_\ell R L\dot\alpha\sin\theta\,\dot\theta$

$\ominus$ $\frac{\partial\mathcal{L}}{\partial\theta} = -M_\ell R L\dot\alpha\dot\theta\sin\theta + M_\ell g L\sin\theta$

$\circleddash$ $(J_\ell + M_\ell L^2)\ddot\theta + M_\ell R L\ddot\alpha\cos\theta - \cancel{M_\ell R L\dot\alpha\dot\theta\sin\theta} + \cancel{M_\ell R L\dot\alpha\dot\theta}\sin\theta - M_\ell g L\sin\theta = 0$

$$\boxed{(J_\ell + M_\ell L^2)\ddot\theta + M_\ell R L\ddot\alpha\cos\theta - M_\ell g L\sin\theta = 0}$$

$F_r = C_r(M_\ell + M_w)g$

$\frac{\delta W}{\delta\alpha} = C_r = F_r\cdot R = R C_r(M_\ell + M_w)g$

- $\frac{\partial\mathcal{L}}{\partial\dot\alpha} = (2J_w + 2M_w R^2 + M_\ell R^2)\dot\alpha + M_\ell R L\dot\theta\cos\theta$

$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot\alpha}\right) = (2J_w + 2M_w R^2 + M_\ell R^2)\ddot\alpha + M_\ell R L\ddot\theta\cos\theta - M_\ell R L\dot\theta^2\sin\theta$

$\ominus$ $\frac{\partial\mathcal{L}}{\partial\alpha} = 0$

$\circleddash$
$$\boxed{(2J_w + 2M_w R^2 + M_\ell R^2)\ddot\alpha + M_\ell R L\ddot\theta\cos\theta - M_\ell R L\dot\theta^2\sin\theta = C - R C_r g(M_\ell + M_w)}$$

couple du moteur sur la roue

couple