# Control of synchronous motor

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# Objective

- Two sessions for presenting the philosophy of control methods for permanent magnet synchronous motors used in several important applications.

- **References:**


  - [Aimé] . Martin Aimé. *"machines synchrones et moteurs brushless"*. Lecture 4AE-SE.
  - [Tounsi]. Patrick Tounsi. " ". Lecture 4AE-SE.
  - [Barret].Philippe Barret. *"Régimes transitoires des machines tournantes électriques"*. Eyrolles 1982.
  - [Gies]. Valentin Gies. http://www.vgies.com/seatech/mecatronique-seatech-isen-5a/
  - [Retif]. J.M. Retif. *"Commande vectorielle des machines synchrones et asynchrones"*. Polycopié, 5ième année GE, Option ISIP INSA Lyon.

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# 1 - Introduction

- Many applications involve synchronous or asynchronous machines (power generation, electric traction etc.)

- Depending of the adopted assumptions, there exist several models for such machines:

- Behn-Eschenbourg model (non saturated machines and non-salient poles)
- Potier model (saturated machines)
- Blondel model (salient poles machines)

- These models are valid in steady-state. They are not really well suited for control purposes.

- This course presents the Park's model for permanent magnet synchronous motors (PMSM) adapted for control purpose, in particular vector control methods (VC). Scalar and vector based control methods are also presented.

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# A magnet in a magnetic field

- Consider a magnet at rest whose magnetic moment is $\overrightarrow{M}$
- There is an ambient magnetic field $\overrightarrow{B}$.
- The magnet undergoes a magnetic torque expressed as $\overrightarrow{C_m} = \overrightarrow{M} \wedge \overrightarrow{B} = -MB\sin\theta_s.\overrightarrow{y}$
- If $\theta_s(t) = \omega_s t$, the magnet being initially at rest, the magnet remains at rest because the average torque $< C_m >= 0$
- If the magnet also rotates at velocity $\theta_s(t) = \omega_s t - \xi$,, then $\overrightarrow{C_m} = -MB\sin\xi.\overrightarrow{y}$ and due to the torque it will maintain its motion
- Remark that the maximal torque corresponds to $\xi = \frac{\pi}{2}$. If $\xi > \frac{\pi}{2}$, the rotation of the magnet stops.

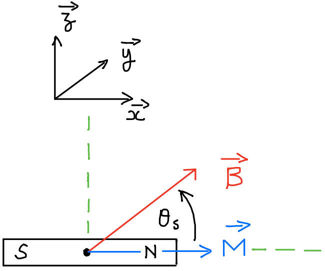


*Fig. 1: Magnet in a magnetic field*

# How a rotating field can be generated?

It is possible to generate a rotating field with a three-phase windings. The current through each coil is time-sinusoidal and $\frac{2\pi}{3}$ out of phase with the other coils, that is

$$i_a(t) = I_s \cos(\omega_s t)$$
$$i_b(t) = I_s \cos(\omega_s t - \frac{2\pi}{3})$$
$$i_c(t) = I_s \cos(\omega_s t - \frac{4\pi}{3})$$

Remark that the system is balanced $i_a(t) + i_b(t) + i_c(t) = 0$ and the associated magnetomotive forces (Ampere) in the rotor direction are

$$F_a(t) = N.i_a(t) \cos \theta_r$$
$$F_b(t) = N.i_b(t) \cos(\theta_r - \frac{2\pi}{3})$$
$$F_c(t) = N.i_c(t) \cos(\theta_r - \frac{4\pi}{3})$$

# How a rotating field can be generated?

Then the resulting magnetomotive force is given by

$$F(t,\theta_r) = \frac{3}{2} N I_s \cos(\omega_s t - \theta_r)$$

leading to a rotating field at $\omega_s$ rad/s.

*Vid 1. A rotating field*

*Vid 2. Synchronous motor*

*Vid 3. Decrochage*

# The permanent magnet synchronous motor

### Main Assumptions

- The magnetic circuit is not saturated. Then the fluxes can be considered as linear functions of currents
- The skin effect can be neglected. Then the current density can be considered uniform in the section of conductors
- The distribution of the magnetomotive force of each phase is sinusoidal, i.e. Only its fundamental harmonic is considered

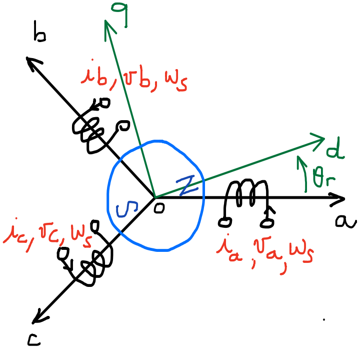


*Fig 2. Synchronous motor*

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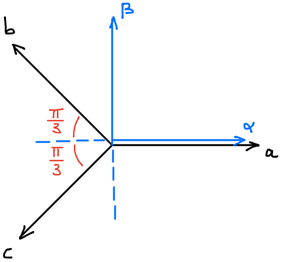
# A first transformation : Clarke-Concordia's transformation

A two-phase system in quadrature with a zero sequence component can be obtained from a three phase system through the following tranformation

$$\begin{bmatrix} g_0 \\ g_\alpha \\ g_\beta \end{bmatrix} = K \underbrace{\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}}_{C} \begin{bmatrix} g_a \\ g_b \\ g_c \end{bmatrix}$$

The inverse transformation is given by

$$\begin{bmatrix} g_a \\ g_b \\ g_c \end{bmatrix} = \frac{1}{K} \begin{bmatrix} \frac{\sqrt{2}}{3} & \frac{2}{3} & 0 \\ \frac{\sqrt{2}}{3} & -\frac{1}{3} & \frac{1}{\sqrt{3}} \\ \frac{\sqrt{2}}{3} & -\frac{1}{3} & -\frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} g_0 \\ g_\alpha \\ g_\beta \end{bmatrix}$$




$$\begin{cases} g_\alpha = g_a - \frac{1}{2} g_b - \frac{1}{2} g_c \\ g_\beta = 0 + \frac{\sqrt{3}}{2} g_b - \frac{\sqrt{3}}{2} g_c \\ g_0 = \frac{1}{\sqrt{2}} g_a + \frac{1}{\sqrt{2}} g_b + \frac{1}{\sqrt{2}} g_c \end{cases}$$

*Fig 3. $\alpha$-$\beta$ transformation*

# A first transformation : Clarke-Concordia's transformation

## REMARKS

### REMARKS

- If $K = \sqrt{\frac{2}{3}}$ the transformation is orthogonal and preserves the value of power. In that case, the transformation is the Concordia's transformation.
- If $K = \frac{2}{3}$ and we change in the first line of the matrix transformation $1/\sqrt{2}$ by $1/2$, the values of currents, voltages and fluxes are preserved, but not the value of power. In that case, the transformation is the Clarke's transformation.

The implications associated with the choice of the value of $K$ are discussed in [Retif]

# A first transformation : Clarke-Concordia's transformation

- Applying the transformation to a three-phase vector of magnitudes $\left[\begin{array}{ccc} g_a & g_b & g_c \end{array}\right]^T$, for example the currents, we obtain

$$\left[\begin{array}{c} i_a \\ i_\alpha \\ i_\beta \end{array}\right] = \left[\begin{array}{c} 0 \\ 3/2KI_s\cos(\omega_s t) \\ 3/2KI_s\sin(\omega_s t) \end{array}\right]$$

- The three-phase windings are equivalent to a two-phase non inductive-coupled windings. From the previous analysis, the magnetic moments $(A.m^2)$ due to the windings are

$$\overrightarrow{M}_\alpha = \frac{3}{2}KSI_s\cos(\omega_s t)\overrightarrow{\alpha},\ \overrightarrow{M}_\beta = \frac{3}{2}KSI_s\sin(\omega_s t)\overrightarrow{\beta}$$

$$\overrightarrow{B} = B_{\max}(\cos\theta_r\overrightarrow{\alpha} + \sin\theta_r\overrightarrow{\beta})$$

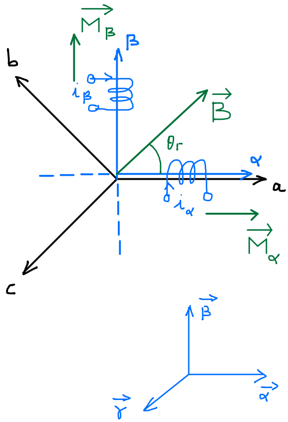



*Fig 4. α-β transformation*

# A first transformation : Clarke-Concordia's transformation

- The resulting torque is

$$\begin{aligned}\overrightarrow{C}_m &= \left(\overrightarrow{M}_\alpha + \overrightarrow{M}_\beta\right) \wedge \overrightarrow{B} \\ &= \overrightarrow{M}_\alpha \wedge \overrightarrow{B} + \overrightarrow{M}_\beta \wedge \overrightarrow{B} \\ &= \frac{3}{2} K S I_s B_{\max} \left[\cos(\omega_s t) \sin\theta_r . \left(\overrightarrow{\alpha} \wedge \overrightarrow{\beta}\right) + \sin(\omega_s t) \cos\theta_r . \left(\overrightarrow{\beta} \wedge \overrightarrow{\alpha}\right)\right] \\ &= \frac{3}{2} K S I_s B_{\max} \sin(\theta_r - \omega_s t) . \underbrace{\overrightarrow{\alpha} \wedge \overrightarrow{\beta}}_{\overrightarrow{\gamma}}\end{aligned}$$

- If $\theta_r(t) = \omega_s t - \xi$, then the norm of the torque becomes

$$C_m = \frac{3}{2} K S I_s B_{\max} \sin\xi = \frac{3}{2} I_s \Phi_{\max} \sin\xi$$

- The use of Clarke-Concordia's transformation simplifies the analysis, but the inductive coupling between the rotor and $\alpha\beta$-windings is a function of $\theta_r$.

We can drop this dependence by introducing another transformation

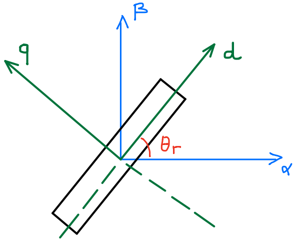
# A second transformation : A simple rotation

The idea is to introduce a rotation of axis $\overrightarrow{\gamma}$, that is

$$\begin{bmatrix} g_0 \\ g_d \\ g_q \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_r & \sin\theta_r \\ 0 & -\sin\theta_r & \cos\theta_r \end{bmatrix}}_{R(\theta_r)} \begin{bmatrix} g_0 \\ g_\alpha \\ g_\beta \end{bmatrix}$$

and

$$\begin{bmatrix} g_0 \\ g_d \\ g_q \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_r & -\sin\theta_r \\ 0 & \sin\theta_r & \cos\theta_r \end{bmatrix}}_{[R(\theta_r)]^{-1}=R(\theta_r)^T} \begin{bmatrix} g_0 \\ g_\alpha \\ g_\beta \end{bmatrix}$$




$$\begin{cases} g_d = \cos\theta_r \, g_\alpha + \sin\theta_r \, g_\beta \\ g_q = -\sin\theta_r \, g_\alpha + \cos\theta_r \, g_\beta \end{cases}$$

*Fig 3. A simple rotation*

# Park's transformation

The new $0dq$-reference is attached to the rotor (axis d in the direction of the rotor magnetic moment). Combining the previous transformations we obtain

$$\begin{bmatrix} g_0 \\ g_d \\ g_q \end{bmatrix} = K \underbrace{\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\theta_r & \underbrace{\frac{\sqrt{3}}{2}}_{\sin\frac{2\pi}{3}} \sin\theta_r - \underbrace{\frac{1}{2}}_{\cos\frac{2\pi}{3}} \cos\theta_r & \underbrace{-\frac{1}{2}}_{\cos\frac{4\pi}{3}} \cos\theta_r - \underbrace{\frac{\sqrt{3}}{2}}_{\sin\frac{4\pi}{3}} \sin\theta_r \\ -\sin\theta_r & \underbrace{\frac{1}{2}}_{-\cos\frac{2\pi}{3}} \sin\theta_r + \underbrace{\frac{\sqrt{3}}{2}}_{\sin\frac{2\pi}{3}} \cos\theta_r & \underbrace{\frac{1}{2}}_{-\cos\frac{4\pi}{3}} \sin\theta_r - \underbrace{\frac{\sqrt{3}}{2}}_{-\sin\frac{4\pi}{3}} \cos\theta_r \end{bmatrix}}_{R(\theta_r).C} \begin{bmatrix} g_a \\ g_b \\ g_c \end{bmatrix}$$

$$= K \underbrace{\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r - \frac{4\pi}{3}) \\ -\sin\theta_r & -\sin(\theta_r - \frac{2\pi}{3}) & -\sin(\theta_r - \frac{4\pi}{3}) \end{bmatrix}}_{P(\theta_r)} \begin{bmatrix} g_a \\ g_b \\ g_c \end{bmatrix}$$

## REMARKS

- This last transformation is the Park's transformation.
- Depending of the values of $K$ and the coefficients of the first line of the matrix transformation ($1/\sqrt{2}$ or $1/2$), it preserves some specific quantities (power, currents, voltages, fluxes...). For a discussion about the differences and interest of the different values of $K$, see [Barret].
- The original value in the Park's article is $K = 2/3$ and the coefficients of the first line $1/2$.

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# Model of a PMSM in the *abc*-Frame

- We consider a synchronous machine with $p$ number of pole-pairs
- The rotor angular speed is $\omega_r = p\Omega_r$ where $\Omega_r$ is its mechanical angular speed, see [Aimé].

**Fluxes**

$$\Phi_{abc} = \begin{bmatrix} \Phi_a \\ \Phi_b \\ \Phi_c \end{bmatrix} = \underbrace{\overbrace{\begin{bmatrix} L & M & M \\ M & L & M \\ M & M & M \end{bmatrix}}^{L_{abc}} \overbrace{\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}}^{i_{abc}}}_{\text{Term due to the stator } \Phi_s} + \underbrace{\Phi_f \begin{bmatrix} \cos\theta_r \\ \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r - \frac{4\pi}{3}) \end{bmatrix}}_{\text{Term due to the rotor } \Phi_r}$$

**Voltages**

$$V_{abc} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \underbrace{\begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix}}_{R_{abc}\text{: Stator resistances}} \underbrace{\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}}_{i_{abc}} + \frac{d\Phi_{abc}}{dt}$$

## Instantaneous Torque

$$\begin{aligned} C_m &= \frac{1}{\Omega_r} i_{abc}^T \frac{d\Phi_r}{dt} = \frac{1}{\Omega_r} i_{abc}^T \frac{d\Phi_r}{d\theta_r}\frac{d\theta_r}{dt} = p i_{abc}^T \frac{d\Phi_r}{d\theta_r} \\ &= -p\Phi_f\left( i_a(t)\sin(\theta_r) + i_b(t)\sin(\theta_r - \frac{2\pi}{3}) + i_c(t)\sin(\theta_r - \frac{4\pi}{3})\right) \\ &= \frac{3}{2} p\Phi_f I_s\left(sin(\omega_s t - \theta_r)\right) \end{aligned}$$

## Mechanical Equation

$$J_r \frac{d\Omega_r}{dt} = C_m - C_{res}$$

where $C_{res}$ is the resistive torque and $J$ is the inertia of the rotating masses.

# Model of a PMSM in the $0dq$-Frame

We consider the Park's transformation with $K = \sqrt{2/3}$

We have

$$\underbrace{\begin{bmatrix} v_0 \\ v_d \\ v_q \end{bmatrix}}_{V_{0dq}} = R(\theta_r)V_{abc} \quad \underbrace{\begin{bmatrix} i_0 \\ i_d \\ i_q \end{bmatrix}}_{i_{0dq}} = R(\theta_r)i_{abc} \quad \underbrace{\begin{bmatrix} \Phi_0 \\ \Phi_d \\ \Phi_q \end{bmatrix}}_{\Phi_{0dq}} = R(\theta_r)\Phi_{abc}$$

$$\Phi_{0dq} = R(\theta_r)\Phi_{abc} = \begin{bmatrix} \underbrace{L+2M}_{L_0} & 0 & 0 \\ 0 & \underbrace{L-M}_{L_d} & 0 \\ 0 & 0 & \underbrace{L-M}_{L_q} \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ \Phi_f \\ 0 \end{bmatrix}$$

# Model of a PMSM in the $0dq$-Frame

- From

$$V_{abc} = R_{abc} i_{abc} + \frac{d\Phi_{abc}}{dt}$$

we deduce

$$\begin{aligned} V_{0dq} = R(\theta_r) V_{abc} &= R(\theta_r) R_{abc} R(\theta_r)^{-1} i_{0dq} + R(\theta_r) \frac{d(R(\theta_r)^{-1} \Phi_{0dq})}{dt} \\ &= R_{abc} i_{0dq} + R(\theta_r) \frac{dR(\theta_r)^{-1}}{dt} \Phi_{0dq} + \frac{d\Phi_{0dq}}{dt} \end{aligned}$$

But we have (Show it)

$$R(\theta_r) \frac{dR(\theta_r)^{-1}}{dt} = \frac{d\theta_r}{dt} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

# Model of a PMSM in the $0dq$-Frame

$$
\begin{aligned}
v_0 &= ri_0 + \frac{d\Phi_0}{dt} = 0 \quad \text{(balanced)} \\
v_d &= ri_d + \frac{d\Phi_d}{dt} - \frac{d\theta_r}{dt}\Phi_q \\
v_q &= ri_q + \frac{d\Phi_q}{dt} + \frac{d\theta_r}{dt}\Phi_d
\end{aligned}
$$

We also have

$$
\begin{aligned}
v_0 i_0 + v_d i_d + v_q i_q &= r\left(i_d^2 + i_q^2\right) + \left(\frac{d\Phi_d}{dt} - \frac{d\theta_r}{dt}\Phi_q\right) i_d + \left(\frac{d\Phi_q}{dt} + \frac{d\theta_r}{dt}\Phi_d\right) i_q \\
&= \underbrace{r\left(i_d^2 + i_q^2\right)}_{\text{Joule losses}} + \underbrace{\left(\frac{d\Phi_d}{dt} i_d + \frac{d\Phi_q}{dt} i_q\right)}_{\text{Reactive Power } P_R} + \underbrace{\left(\frac{d\theta_r}{dt}\Phi_d i_q - \frac{d\theta_r}{dt}\Phi_q i_d\right)}_{\text{Active Power } P_A}
\end{aligned}
$$

# Model of a PMSM in the 0*dq*-Frame

The torque is given by

$$\begin{aligned} C_m = \frac{P_A}{\Omega_r} &= p\left(\Phi_d i_q - \Phi_q i_d\right) \\ &= p\left(\left(L_d i_d + \Phi_f\right) i_q + L_q i_q i_d\right) \\ &= p\left(\Phi_f i_q + (L_d - L_q) i_d i_q\right) \\ &= p\, \Phi_f\, i_q \end{aligned}$$

for a no salient poles machine because $L_q = L_d$

# Model of a PMSM in the $0dq$-Frame

**Fluxes**

$$\Phi_0 = L_0 i_0, \qquad L_0 = L + 2M$$
$$\Phi_d = L_d i_d + \Phi_f, \qquad L_d = L - M$$
$$\Phi_q = L_q i_q, \qquad L_q = L - M$$

**Voltages**

$$v_0 = r i_0 + L_0 \frac{di_0}{dt}$$
$$v_d = r i_d + L_d \frac{di_d}{dt} - \omega_r L_q i_q$$
$$v_q = r i_q + L_q \frac{di_q}{dt} + \omega_r L_d i_d + \omega_r \Phi_f, \qquad \omega_r = \frac{d\theta_r}{dt}$$

**Instantaneous Torque**

$$C_m = p\Phi_f i_q = \frac{3}{2} p \Phi_f I_s \left(sin(\omega_s t - \theta_r)\right)$$

**Mechanical Equation**

$$J_r \frac{d\Omega_r}{dt} = C_m - C_{res}$$

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# An overview

An overview of the main speed control strategies is proposed below

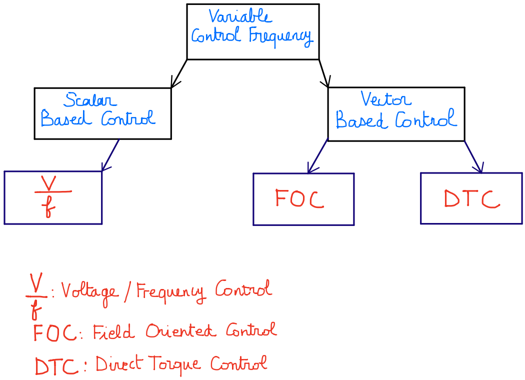


*Fig 4. An overview*

# Scalar based control ($V/f$)

The active power transmitted to the stator can be expressed as (balanced system)

$$P_A = v_a i_a + v_b i_b + v_c i_c = 3 V_s I_s \cos\varphi$$

If we neglect all the losses and suppose that all the power is transmitted to the rotor, the torque is given by

$$C_m = \frac{P_A}{\Omega_r} = 3p\frac{V_s}{\omega_r} I_s \cos\varphi$$

The idea is to keep stator flux constant at a rated value over the entire speed range which leads to a constant ratio $\frac{V_s}{\omega_r}$ . Supposing the machine in steady steady state, we can deduce that

$$v_d = r i_d - \omega_r \Phi_q \qquad v_q = r i_q + \omega_r \Phi_d$$

If the rotor speed is high, the resistive drop voltage can be neglected and

$$\Phi_q = -\frac{v_d}{\omega_r} \qquad \Phi_d = \frac{v_q}{\omega_r} \qquad V_s = \sqrt{v_d^2 + v_d^2}$$

We can recover the corresponding values of $v_a$, $v_b$ and $v_c$ by the inverse Park's transformation

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & -\sin\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r - \frac{4\pi}{3}\right) & -\sin\left(\theta_r - \frac{4\pi}{3}\right) \end{bmatrix} \begin{bmatrix} v_d \\ v_q \end{bmatrix}$$

For small speeds, it can be necessary to compensate the resistive voltage drop.

The principle is summarized in the following figure

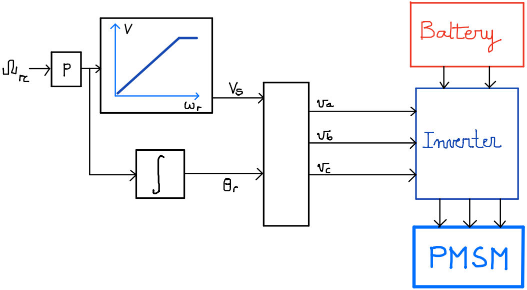


*Fig 4. Scalar Based Control*

## Advantages of Scalar Based Control

- The implementation is simple
- The control $V/f$ being an open loop strategy, no sensor is needed

## Drawbacks of Scalar Based Control

- The transient regime is not really managed
- The performances can be mediocre

# Vector Based Control (FOC)

From the expression of the torque $C_m = p\Phi_f i_q$, the idea is to regulate $i_d$ around 0 and control the torque through $i_q$.

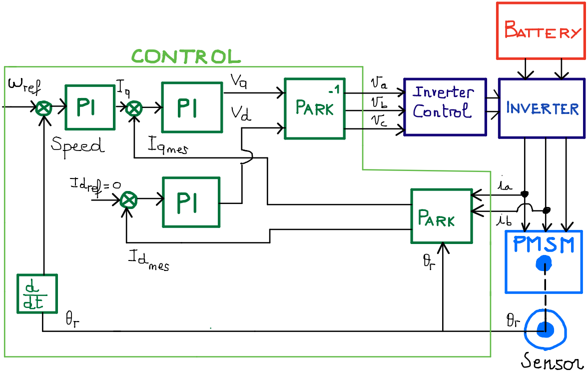


*Fig 5. Vector Based Control*

## Advantages of Vector Based Control

- Efficient for regulating the motor speed with a good precision
- The performances of a synchronous motor controlled using this technique are superior to a DC motor of the same power (starting torque, precision, dynamic response...)

## Drawbacks of Vector Based Control

- The control is complex and some sensors are needed
- The control must be implemented using a microcontroller (DSP,...)